MATHEMATICS PREPARATION 10

Teacher Resource Manual

Alberta LEARNING
Learning and Teaching Resources Branch

MATHEMATICS 10

PREPARATION

Teacher Resource Manual

Alberta
LEARNING
Learning and Teaching Resources Branch

Mathematics Preparation 10

Teacher Resource Manual

2000

Alberta Learning Cataloguing in Publication Data

Alberta. Alberta Learning. Learning and Teaching Resources Branch.
Mathematics Preparation 10 : teacher resource manual.

Also available on the Internet: <http://www.learning.gov.ab.ca>.

ISBN 0–7785–1105–7

1. Mathematics — Study and teaching (Secondary) — Alberta.
I. Title.

QA14.C22A3.A333 2000 372.7

Additional copies are available for purchase from:

Learning Resources Distributing Centre
12360 – 142 Street
Edmonton, Alberta T5L 4X9
Canada

The primary intended audience for this document is:

Administrators	
Counsellors	
General Audience	
Parents	
Students	
Teachers	✓

Every effort has been made to provide proper acknowledgement of original sources and to comply with copyright law. If cases are identified where this has not been done, please notify Alberta Learning by writing to:

Director, Learning and Teaching Resources Branch
Alberta Learning
Devonian Building
11160 Jasper Avenue
Edmonton, Alberta T5K 0L2
Canada

ACKNOWLEDGEMENTS

Alberta Learning wishes to thank the British Columbia Ministry of Education, Manitoba Education and Training, and the Nova Scotia Department of Education and Culture for the sharing of resources. Alberta Learning also expresses its appreciation to the following individuals who contributed to the development of this manual.

Marilyn Blackhall	Calgary Roman Catholic Separate School District No. 1
George Dashkewytch	High Prairie School Division No. 48
Don Deeter	Rocky View School Division No. 41
Patti Edwards	Calgary School District No. 19
Kathy Goudreau	Edmonton School District No. 7
Metro Huculak	St. Thomas Aquinas Roman Catholic Separate Regional Division No. 38
François Lizaire	French Language Services Branch, Alberta Learning
Marilyn Shannon	Norquest College
Cathy Steele	Calgary Roman Catholic Separate School District No. 1
Reed Svenson	Pembina Hills Regional Division No. 7

Alberta Learning

Gina Vivone-Vernon	Director, Learning and Teaching Resources Branch
Debbie Duvall	Learning and Teaching Resources Branch
Katie Pallos-Haden	Curriculum Branch
Jack Edwards	Learning and Teaching Resources Branch
Chris Ewanchuk	Learning and Teaching Resources Branch
Kim Blevins	Learning and Teaching Resources Branch
Lin Hallett	Learning and Teaching Resources Branch
Dianne Moyer	Learning and Teaching Resources Branch
Esther Yong	Learning and Teaching Resources Branch

TABLE OF CONTENTS

PURPOSE OF THE TEACHER RESOURCE MANUAL

This Teacher Resource Manual (TRM) is designed to help teachers implement the Mathematics Preparation 10 course. The TRM is a support document that provides helpful information to classroom teachers. The instructional and assessment strategies presented are suggestions only and are not legally prescribed. The general and specific outcomes in the program of studies for Mathematics Preparation 10 are included in this manual to assist teachers. These outcomes have been shaded to show that they are prescriptive in nature.

The Rationale, Philosophy, Student Expectations, Instructional Focus, Horizontal Correlation, and High School Course Structure from the program of studies for Mathematics Preparation 10 are also shaded and included on pages 3–14 of this manual.

A complete copy of the program of studies for Mathematics Preparation 10 is available at the Alberta Learning web site at <http://www.learning.gov.ab.ca>.

HOW TO USE THE TEACHER RESOURCE MANUAL

The Mathematics Preparation 10 TRM is intended to help teachers translate the Mathematics Preparation 10 program of studies into classroom practice. The design of the manual is as follows.

STRAND—Lists the strand and substrand as per the program of studies.

GENERAL OUTCOME—This section lists the general outcome from the program of studies.

SPECIFIC OUTCOME—This section identifies the specific outcome by number and in words.

MANIPULATIVES—Manipulatives used in the activities provided for the specific outcome are listed here. This list is not exhaustive, as many other manipulatives may be used in addressing the outcomes.

SUGGESTED LEARNING RESOURCES—Resources that are commonly found in Alberta schools have been identified for each specific outcome. Included are resources that are currently authorized for Alberta mathematics courses and resources that are no longer authorized but may still be available in schools. Teachers are encouraged to use current authorized resources whenever these are available.

TECHNOLOGY CONNECTIONS—Any technology required to carry out the suggested activities for the specific outcome are listed here. Students should be able to complete the Mathematics Preparation 10 course using a scientific calculator. However, if a teacher wishes to include the use of graphing calculators in the course, activities to support this have been included, where appropriate. The decision to use a graphing calculator lies with the teacher. It is not a requirement of this course. **Note**: The list of technology connections cited in this document is not exhaustive. There are many computer programs, web sites and other technology connections that would be useful in addressing the outcomes. This list only cites technology that is used in the activities provided for the specific outcome.

INSTRUCTIONAL STRATEGIES/SUGGESTIONS—These are ideas for instruction and sample student activities. This list is not intended to be comprehensive enough to provide all activities required to teach the course. It does, however, provide sample activities to help guide the teacher in developing further teaching activities.

Teaching Notes—These are suggestions for the teacher that other teachers have found helpful. This section also provides a place for teachers to write notes to themselves for the next year.

TASKS FOR INSTRUCTION AND/OR ASSESSMENT—A variety of assessment approaches should be used to collect information about student learning. These assessment samples have been provided to model broad-based assessment. Again, the samples included are not intended to be comprehensive but rather to provide ideas upon which the teacher can build.

PROGRAM RATIONALE AND PHILOSOPHY

Rationale

Mathematics is a common human activity, increasing in importance in a rapidly advancing, technological society. A greater proficiency in using mathematics increases the opportunities available to individuals. Students need to become mathematically literate in order to explore problem-solving situations, accommodate changing conditions, and actively create new knowledge in striving for self-fulfillment.

A significant proportion of Grade 9 Mathematics students have marks that are close to, or slightly under, the pass mark of 50 per cent. These students are not sufficiently prepared to succeed in either Applied Mathematics 10 or Pure Mathematics 10 in senior high school. Nor does the alternative of Mathematics 14–24 address their needs. A senior high school mathematics course is needed to meet the needs of Grade 10 students who wish to enroll in either Applied Mathematics 10 or Pure Mathematics 10, but do not possess the prerequisite skills.

Students should not have to repeat outcomes for which they have already achieved an acceptable standard. Because of the sequential development of mathematical skills, some students will need to begin their programs with Grade 7 or Grade 8 outcomes. Others may only require preparation in some of the Grade 9 Mathematics outcomes. Variable credits allows for this flexibility. Schools can offer and students can choose from 1 to 5 credits for Mathematics Preparation 10 depending on the needs of the student.

Philosophy

A senior high school mathematics course that simply repeats the outcomes of Grade 9 Mathematics is not the optimal solution for Grade 10 students who have not met the standards of junior high mathematics. These students require a course that will address their individual needs with a focus on the mathematics skills and knowledge they lack from junior high school mathematics, or earlier. Students entering Grade 10 need a repertoire of fundamental mathematics skills and concepts. They also need to understand the ideas that make up those concepts and how they are related.

Students are required to demonstrate effective communication skills. When accomplishing program outcomes, students will be expected to explain, to illustrate, to reason and to make connections. Multiple solution strategies to problems and problem contexts will be expected as students work through routine and nonroutine problems.

Technology is an integral part of this mathematics program. Calculators and computer programs are used to enhance conceptual understanding and to facilitate higher order thinking through exploration, modeling and problem solving.

Students learn by attaching meaning to what they do; and they must be able to construct their own meaning of mathematics. This meaning is best developed when learners encounter mathematical experiences that proceed from the simple to the complex and from the concrete to the abstract. The use of manipulatives can address the diversity of learning styles and developmental stages of students and can enhance the formation of sound, transferable, mathematical concepts. At all levels, students benefit from working with appropriate materials, tools and contexts when constructing personal meaning about new mathematical ideas. The learning environment should value and respect each student's way of thinking, so that the learner feels comfortable in taking intellectual risks, asking questions and posing conjectures.

Student Expectations

It is important for students to develop a positive attitude toward mathematics so they can become confident in their ability to use mathematics to solve real-life problems. Students should receive a level of mathematics education appropriate to their abilities and needs. At the completion of this program, students must have the mathematical knowledge, skills and attitudes needed to allow them to succeed in high school mathematics and become mathematically literate adults.

The program of studies incorporates the seven interrelated mathematical processes that are intended to permeate teaching and learning.

The students are expected to:

- *Communication [C]* – communicate mathematically
- *Connections [CN]* – connect mathematical ideas to other concepts in mathematics, to everyday experiences and to other disciplines
- *Estimation and Mental Mathematics [E]* – use estimation and mental mathematics where appropriate
- *Problem Solving [PS]* – relate and apply new mathematical knowledge through problem solving
- *Reasoning [R]* – reason and justify their thinking
- *Technology [T]* – select and use appropriate technologies as tools to solve problems
- *Visualization [V]* – use visualization to assist in processing information, making connections and solving problems.

For more details on these mathematical processes, please refer to the *Alberta Program of Studies for K–9 Mathematics*, June 1996, pages 6–11.

Instructional Focus

Students are curious, active learners who have individual interests, abilities and needs. They come to classrooms with different knowledge, life experiences and backgrounds that generate a range of attitudes about mathematics and life. Course delivery must be commensurate with differing abilities, interests and learning styles and be designed to enable students to have success in mathematics. The use of projects and technology should be emphasized to facilitate entrance into Applied Mathematics 10.

Several additional considerations are important.

- The use of diagnostic strategies is encouraged to determine a student's area of strength and weakness. From this, individualized instruction should emerge.
- Concepts should be introduced, using manipulatives, and gradually developed from the concrete to the pictorial to the symbolic.
- Problem solving, reasoning and connections are vital to increasing mathematical power and must be integrated throughout the program. Activities that take place in the classroom should stem from a problem-solving approach with connections to the real world whenever possible. A minimum of half the available time within all strands could be dedicated to activities related to these processes.
- In mastering basic facts and arithmetic operations, the emphasis should be on developing understanding of the concept/processes, not on rote drill and practice. Although drill and practice needs to be part of a mathematics program, it should occur after students have developed an understanding of the concept/process and should not occur in isolation. If students understand the process, the amount of time needed for drill and practice is significantly reduced.
- There is to be a balance between estimation and mental mathematics, paper and pencil exercises and the appropriate use of technology, including calculators and computers.
- By decreasing emphasis on the size of numbers used in paper and pencil calculations, more time is available for concept development.
- There is an assumption made that all students have regular access to appropriate technology. For Mathematics Preparation 10, the scientific calculator and standard spreadsheet programs are appropriate.

Horizontal Correlation

The General Outcomes from Mathematics Preparation 10 have been correlated to the corresponding General Outcomes in Grade 6 through Grade 10. These are shown in the chart below:

GENERAL OUTCOMES—Number Strand

Substrand	Grade 6	Grade 7	Grade 8
Number Concepts *Students will:* • use numbers to describe quantities • represent numbers in multiple ways.	Develop a number sense for decimals and common fractions, explore integers, and show number sense for whole numbers.	Demonstrate a number sense for decimals and integers, including whole numbers.	Demonstrate a number sense for rational numbers, including common fractions, integers and whole numbers.
Number Operations *Students will:* • demonstrate an understanding of and proficiency with calculations • decide which arithmetic operation or operations can be used to solve a problem and then solve the problem.	Apply arithmetic operations on whole numbers and decimals in solving problems.	Apply arithmetic operations on decimals and integers, and illustrate their use in solving problems. Illustrate the use of rates, ratios, percentages and decimals in solving problems.	Apply arithmetic operations on rational numbers to solve problems. Apply the concepts of rate, ratio, percentage and proportion to solve problems in meaningful contexts.

Grade 9	*Mathematics Preparation 10*	Grade 10
Explain and illustrate the structure and the interrelationship of the sets of numbers within the rational number system. Develop a number sense of powers with integral exponents and rational bases.	*Demonstrate a knowledge of the interrelationships of the sets of numbers within the real number system.* *Develop a number sense of powers with integral exponents and rational bases.*	Analyze the numerical data in a table for trends, patterns and interrelationships. Explain and illustrate the structure and the interrelationship of the sets of numbers within the real number system.
Use a scientific calculator or a computer to solve problems involving rational numbers. Explain how exponents can be used to bring meaning to large and small numbers, and use calculators or computers to perform calculations involving these numbers.	*Decide which arithmetic operations can be used to solve problems and then solve the problem.* *Illustrate and apply the concepts of rates, ratios, percentages and proportion to solve problems.*	Use basic arithmetic operations on real numbers to solve problems. Describe and apply arithmetic operations on tables to solve problems, using technology as required. Use exact values, arithmetic operations and algebraic operations on real numbers to solve problems.

GENERAL OUTCOMES—Patterns and Relations Strand

Substrand	Grade 6	Grade 7	Grade 8
Patterns *Students will:* • use patterns to describe the world and to solve problems.	Use relationships to summarize, generalize and extend patterns, including those found in music and art.	Express patterns, including those used in business and industry, in terms of variables, and use expressions containing variables to make predictions.	Use patterns, variables and expressions, together with their graphs, to solve problems.
Variables and Equations *Students will:* • represent algebraic expressions in multiple ways.	Use informal and concrete representations of equality and operations on equality to solve problems.	Use variables and equations to express, summarize and apply relationships as problem-solving tools in a restricted range of contexts.	Solve and verify one-step and two-step linear equations with rational number solutions.

Grade 9	*Mathematics Preparation 10*	Grade 10
Generalize, design and justify mathematical procedures, using appropriate patterns, models and technology.	*Generalize, design and justify mathematical procedures, using appropriate patterns and technology.*	Generate and analyze number patterns.
Solve and verify linear equations and inequalities in one variable. Generalize arithmetic operations from the set of rational numbers to the set of polynomials.	*Solve and verify linear equations and inequalities using one variable.* *Generalize arithmetic operations from the set of rational numbers to the set of polynomials.*	Generalize operations on polynomials to include rational expressions.

GENERAL OUTCOMES—Shape and Space Strand

Substrand	Grade 6	Grade 7	Grade 8
Measurement *Students will:* • describe and compare everyday phenomena, using either direct or indirect measurement.	Solve problems involving perimeter, area, surface area, volume and angle measurement.	Solve problems involving the properties of circles and their connections with angles and time zones.	Apply indirect measurement procedures to solve problems. Generalize measurement patterns and procedures, and solve problems involving area, perimeter, surface area and volume.
3-D Objects and 2-D Shapes *Students will:* • describe the characteristics of 3-D objects and 2-D shapes, and analyze the relationships among them.	Use visualization and symmetry to solve problems involving classification and sketching.	Link angle measures to the properties of parallel lines.	Link angle measures and the properties of parallel lines to the classification and properties of quadrilaterals.
Transformation *Students will:* • perform, analyze and create transformations.	Create patterns and designs that incorporate symmetry, tessellations, translations and reflections.	Create and analyze patterns and designs, using congruence, symmetry, translation, rotation and reflection.	Create and analyze design problems and architectural patterns, using the properties of scaling, proportion and networks.

Grade 9	*Mathematics Preparation 10*	Grade 10
Use trigonometric ratios to solve problems involving a right triangle. Describe the effects of dimension changes in related 2-D shapes and 3-D objects in solving problems involving area, perimeter, surface area and volume.	*Solve problems using right angle triangles.* *Solve problems using perimeter, area, surface area and volume.*	Demonstrate an understanding of scale factors, and their interrelationship with the dimensions of similar shapes and objects. Solve problems involving triangles, including those found in 3-D and 2-D applications. Use measuring devices to make estimates and to perform calculations in solving problems.
Specify conditions under which triangles may be similar or congruent, and use these conditions to solve problems. Use spatial problem solving in building, describing and analyzing geometric shapes.	*Specify conditions under which triangles may be similar and use these conditions to solve problems.*	Solve coordinate geometry problems involving lines and line segments.
Apply coordinate geometry and pattern recognition to predict the effects of translations, rotations, reflections and dilatations on 1-D lines and 2-D shapes.	*Create and analyze patterns and design, using symmetry, translation, rotation and reflection.*	

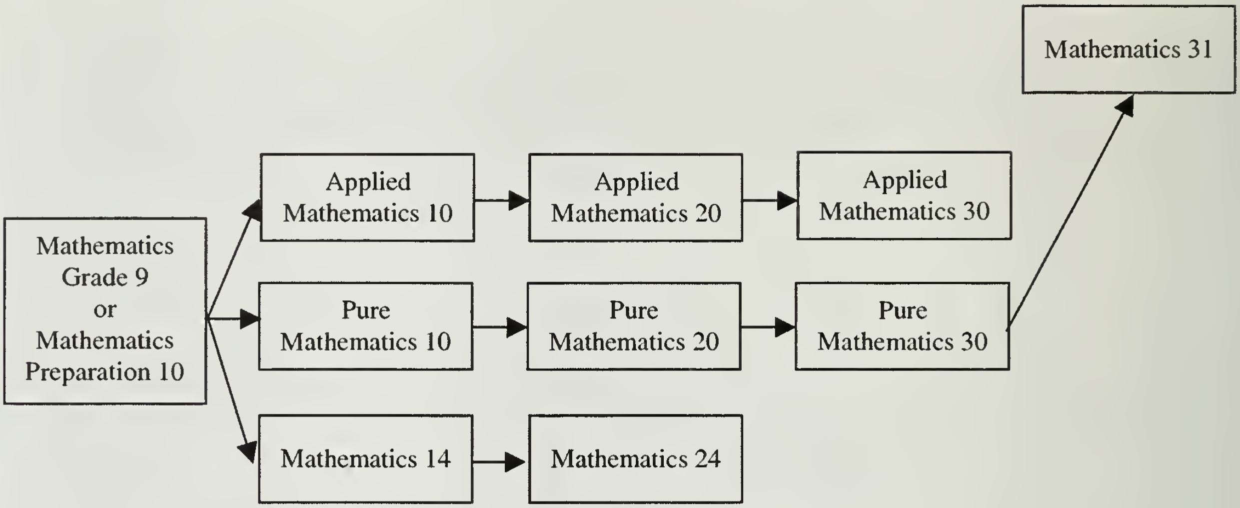
GENERAL OUTCOMES—Statistics and Probability Strand

Substrand	Grade 6	Grade 7	Grade 8
Data Analysis *Students will:* • collect, display and analyze data to make predictions about a population.	Develop and implement a plan for the collection, display and analysis of data gathered from appropriate samples.	Develop and implement a plan for the collection, display and analysis of data, using measures of variability and central tendency.	Develop and implement a plan for the collection, display and analysis of data, using technology, as required. Evaluate and use measures of central tendency and variability.

Grade 9	*Mathematics Preparation 10*	Grade 10
Collect and analyze experimental results expressed in two variables, using technology, as required.	*Develop and implement a plan for the display and analysis of data.* *Analyze experimental results expressed in two variables.*	Implement and analyze sampling procedures, and draw appropriate inferences from the data collected. Apply line-fitting and correlation techniques to analyze experimental results.

High School Course Structure

Upon successful completion of Mathematics Preparation 10 a student may enroll in Applied Mathematics 10 or Pure Mathematics 10. A student who is not successful in Mathematics Preparation 10 may either repeat the course or enroll in Mathematics 14.



STRAND: NUMBER (NUMBER CONCEPTS)

GENERAL OUTCOME

Demonstrate a knowledge of the interrelationship of the sets of numbers within the real number system.

SPECIFIC OUTCOME

1. Compare and order integers. [R, V] (7–12)

MANIPULATIVES

- Number lines
- Thermometer

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Interactions 7*, pp. 88–90
- *Mathpower 7*, pp. 62–63
- *Minds on Math 7*, pp. 262–267
- *Minds on Math 8*, pp. 264–267
- *TLE 7*, Introduction to Integers, Student Refresher pp. 18–19, Teacher's Manual pp. 48–51

Previously Authorized Resources

- *Journeys in Math 8*, pp. 275–277
- *Journeys in Math 9*, pp. 35–37

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes Visual and concrete materials are useful for integer instruction.	Demonstrate to students that many real-life situations involve integers. In order to help students understand integers and their operations, use relevant real-world applications, such as temperature, altitude, banking, games and sports.

1. Write the integer that corresponds to the phrase, or write a phrase that corresponds to the integer.

word phrases	integer
a loss of $5	
move 6 units right	
positive 3	
	$^{-}16$
12° below zero	
	+7
18 m above sea level	
3 under par	
	$^{-}19$
	+212

2. Draw a number line from –10 to +10. Circle –8, –5, +2 and +7 on the number line.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

3. Using a number line, write an integer that satisfies:
   - 4 greater than $^{-}3$
   - 2 less than 0
   - between $^{-}3$ and $+2$
   - 2 units to the left of $^{-}1$
   - 3 units to the right of $^{-}1$

4. Compare the following integers, using "greater than", "less than" and ">", "<".

   Compare:
   - $^{-}4$, 7
   - $^{-}5$, $^{-}17$
   - 3, $^{-}3$
   - 0, $^{-}6$

5. Arrange the following integers from smallest to largest.
   - $^{-}6$, 7, $^{-}1$, 0, 3, 5, $^{-}4$
   - 17, $^{-}21$, 42, $^{-}3$, 92
   - $^{-}29$, 19, $^{-}9$, $^{-}49$, 79

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes Don't use vector quantities, such as velocity, where the negative sign means direction, or scalar quantities, such as mass, where the negative sign is meaningless.	**Performance** 1. Show on a number line why $^{-}3$ is less than $^{-}1$. 2. Use a number line to arrange 4, $^{-}6$, 0, $^{-}2$. **Journal Entry** 1. Explain why a negative number farther left from 0 is smaller than a negative number closer to 0. 2. Explain why a positive number is always greater than a negative number. **Interview** 1. Explain why a golfer likes a negative golf score. 2. Use integers to talk about altitudes. 3. Use integers to talk about bank accounts. **Portfolio** 1. Use newspapers or magazines to find articles that include integers; e.g., golf scores, hockey plus/minus, stock market. **Paper and Pencil** 1. Plot points +5 and $^{-}5$ on a number line. What do you notice about them? Why do you think number pairs such as $^{-}5$ and +5 are called opposites? 2. Write an integer for each of the following situations: a. A person walks up 9 flights of stairs. b. An elevator goes down 7 floors. c. The temperature falls by 17 degrees. d. Sue deposits $150 in the bank. e. The peak of the mountain is 2023 m above sea level.

STRAND: NUMBER (NUMBER CONCEPTS)

GENERAL OUTCOME

Demonstrate a knowledge of the interrelationship of the sets of numbers within the real number system.

SPECIFIC OUTCOME

2. Find and be able to model an understanding of common multiples, common factors, lowest common multiples, greatest common factors as it applies to whole numbers. [C, PS, R, V] (6–4)

MANIPULATIVES

- Timetable grid

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Mathpower 7*, pp. 176–177
- *Mathpower 9*, pp. 182–186
- *Minds on Math 7*, pp. 92–102
- *Minds on Math 8*, p. 154
- *TLE 7*, Fraction Conversions, Student Refresher pp. 10–11, Teacher's Manual pp. 32–35

Previously Authorized Resources
- *Journeys in Math 8*, pp. 134–135
- *Journeys in Math 9*, pp. 10–11

TECHNOLOGY CONNECTIONS

- Scientific calculator
- Graphing calculator (optional)

<table>
<tr><th></th><th>INSTRUCTIONAL STRATEGIES/SUGGESTIONS</th></tr>
<tr><td>Teaching Notes</td><td>
An understanding of common factors and greatest common factors (GCF) will be useful to students in reducing fractions to lowest terms and in problem-solving situations.
It is often useful to pose a problem as a starting point for instruction; e.g.:
Sue's uncle donated 96 juice boxes and 64 chocolate treats for her party. What is the largest number of people that can be at the party and share the food equally (without breaking treats)?
Students worked with factors, common factors and prime numbers in previous grades. It may be necessary to review the terms factor, common factor and prime number before extending to prime factorization.
Adapted with permission from Atlantic Canada Mathematics Curriculum: Grade 7.
1. Have students use the "splitting into equal groups" model of division to work out the answers to the following questions.

$8 \div 2 = ?$
$6 \div 2 = ?$
$4 \div 2 = ?$
$2 \div 2 = ?$
$0 \div 2 = ?$

</td></tr>
</table>

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Have them discuss the last situation: 0 objects to split up with 2 in each group. Then have them consider 2 ÷ 0 (2 objects split up with 0 in each group). Most students will see the question itself as being silly and, therefore, there is no answer. • *Activity*: Building patterns on a number chart Goal: To explore number concepts and their relationships, such as: - primes and composites - multiples and factors - odd and even numbers - divisibility rules for 2, 3, 4, 5, 9 Materials: Each student receives - a number chart 1 to 100 - colour pencils or markers (7 different colours) Student instructions: 1. Place a **yellow** check mark on all multiples of 2. Look at the pattern and describe it. (*Possible student descriptions*: a) Alternate squares are filled. b) It looks like a checkerboard. c) Those are even numbers. They are multiples of 2.) 2. Continue by placing a **red** check mark on all multiples of 3. Look at the pattern and describe it. (*Possible student descriptions*: a) They are even and odd numbers. b) Some numbers have both red and yellow checks. c) Some numbers have two factors.) 3. Continue by placing a **blue** check on all multiples of 4. Look at the pattern and describe it. (*Possible student descriptions*: a) All are even. b) Some numbers have all 3 colours. c) Some numbers have 3 factors.) 4. Continue filling in the pattern in the following way: **purple** — multiples of 5 **green** — multiples of 6 **orange** — multiples of 7 **black** — multiples of 8 Reproduced, by permission, from Manitoba Education and Training. *Grades 5 to 8 Mathematics: A Foundation for Implementation*. Winnipeg, MB: Manitoba Education and Training, 1997.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	

2. Lowest Common Multiple (LCM)

2.1

×	1	2	3	4	5	6	7	8	9	10	11	12
1	1	2	3	4	5	6	7	8	9	10	11	12
2	2	4	6	8	10	12	14	16	18	20	22	24
3	3	6	9	12	15	18	21	24	27	30	33	36
4	4	8	12	16	20	24	28	32	36	40	44	48
5	5	10	15	20	25	30	35	40	45	50	55	60
6	6	12	18	24	30	36	42	48	54	60	66	72
7	7	14	21	28	35	42	49	56	63	70	77	84
8	8	16	24	32	40	48	56	64	72	80	88	96
9	9	18	27	36	45	54	63	72	81	90	99	108
10	10	20	30	40	50	60	70	80	90	100	110	120
11	11	22	33	44	55	66	77	88	99	110	121	132
12	12	24	36	48	60	72	84	96	108	120	132	144

Cut the grid into horizontal strips. To find the LCM for 3 and 5, place the 3 and 5 strips together and check the lowest number common to both strips.

2.2 To find the LCM, use repeated division by prime factors.

```
2 | 24, 40
 × 2 | 12, 20
    × 2 | 6, 10
       ×   3, × 5
2 × 2 × 2 × 3 × 5 = 120
```

```
2 | 12, 10, 9
   3 | 6,  5,  9
       2   5   3
2 × 3 × 2 × 5 × 3 = 180
```

2.3 Use a calculator to generate multiples of a number by keying in the number and the + key repeatedly. **Note**: Different procedures apply to different calculators.

Extension: Using the TI-83 calculator, select
Math → Number
↓
LCM (
↓
enter the 2 numbers separated by a comma. Close bracket. ENTER.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	

2.4 Use a prime factorization.

$24 = 2 \times 2 \times 2 \times 3$
$40 = 2 \times 2 \times 2 \times 5$

↓ ↓ ↓

$\text{LCM} = 2 \times 2 \times 2 \times 3 \times 5$
$= 120$

3. Greatest Common Factor (GCF)

3.1 To find the GCF of 24 and 36 using the listing of factors method, list the factors for each number.

24 = $\boxed{1}$, $\boxed{2}$, $\boxed{3}$, $\boxed{4}$, $\boxed{6}$, 8, $\boxed{12}$, 24
36 = $\boxed{1}$, $\boxed{2}$, $\boxed{3}$, $\boxed{4}$, $\boxed{6}$, 9, $\boxed{12}$, 18, 36

Circle the factors in common and select the greatest common factor, which in this case is 12.

3.2 To find the GCF, use repeated division by prime factors.

```
3 |36,  24
  2 |12,  8
    2 |6,  4
      1 |3,  2
```

$\text{GCF} = 3 \times 2 \times 2 \times 1 = 12$

3.3 The prime factorization method should also be considered in determining the GCF. To find the GCF of 24 and 36 using prime factorization, first write the prime factors for each number:

$24 = \boxed{2} \times \boxed{2} \times 2 \times \boxed{3}$
$36 = \boxed{2} \times \boxed{2} \times 3 \times \boxed{3}$

Then choose factors common to both numbers and multiply them to get the GCF. The GCF of 24 and 36 is $2 \times 2 \times 3 = 12$.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Extension: Using the TI-83 calculator, select Math → Number ↓ GCF (↓ enter the 2 numbers separated by a comma. Close bracket. ENTER. **Application Questions** 1. Three sisters, Janice, Jackie, and Gaye, live in 3 different cities. Janice returns home every 2 years, Jackie returns home every 3 years, and Gaye every 5 years. If the sisters were last reunited in 1990, what year will they get together next? Reproduced, by permission, from Manitoba Education and Training. *Grades 5 to 8 Mathematics: A Foundation for Implementation*. Winnipeg, MB: Manitoba Education and Training, 1997.
Suggestion: Read *Spaghetti and Meatballs for All!: A Mathematical Story* by Marilyn Burns.	2. You are making reservations for a dinner party at a local restaurant. The restaurant can sit the guests at tables of 2, 3, 4 or 5. If all the places at a table must be filled and the maximum number of guests the restaurant can accommodate is 28, what is the maximum number of guests you can invite to the dinner party for each of the above seating arrangements?

Strand: Number (Number Concepts)
Specific Outcome: 2. Find and be able to model an understanding of common multiples, common factors, lowest common multiples, greatest common factor as it applies to whole numbers. [C, PS, R, V] (6–4)

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes Questions should include components that do and do not require calculators.	**Paper and Pencil** [1] 1. Chris is filling loot bags for his little sister's birthday party. He has 24 toys, 36 caramels and 60 chocolate treats. What is the largest number of bags that can be filled if all the treats are to be used, no treats are subdivided, and all children receive the same items in their loot bags? 2. John is creating a miniature, quilted wall hanging made of square blocks. He wants the wall hanging to be exactly 15 cm by 20 cm. a. Find the size of the largest finished square block with whole number side lengths that can be used to exactly cover the area. b. Find the GCF of 15 and 20. c. Compare your answers in part a and part b, and describe what you notice. 3. Sarah decides to make a quilt for her bed based on the design John used in question 2. However, she feels that she should enlarge the block size so that it will not require as many blocks. The bed quilt must be 200 cm by 250 cm. a. Make a list of possible sizes for the square blocks. b. What is the largest square block that can be used? c. Sarah decided that the blocks should be larger than 15 cm but smaller than 30 cm. What size would the finished blocks need to be to make this work? 4. The GCF of 8 and an unknown number is 4. Find three possible values for the missing number. Describe what all the values have in common. 5. Bill and Jenny regularly exercise at a local gymnasium. Bill exercises one day out of every 6, and Jenny exercises one day out of every 4. If they both start today, and the gymnasium is open every day, how many days will they exercise on the same day in a 5 week span? 6. Find the LCM for: a. 9, 12, 15 b. 5, 10, 20 7. Find the GCF for: a. 36, 42, 60 b. 40, 50, 65

[1] Paper and Pencil questions 1 to 4 and 8 to 11 are adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 7.*

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	8. a. The LCM of two numbers is 24. Find all possible values for the numbers. b. The GCF of 8 and an unknown number is 4, and the LCM of the two numbers is 24. Find a possible value for the unknown number. 9. If the GCF of two numbers is 8, and the LCM is 80, what are the possibilities for the pair of numbers? 10. Joe and Pat work part-time at a music rental store. Joe works once every four days, and Pat works once every six days. a. If they both started work on September 27, and the store is open every day, what will be the next date they work together? b. Find two more dates on which they will work together. 11. Solve the following, using a number line. Sarah has three aunts who live in other provinces of Canada. Her aunt who lives in Vancouver visits every fourth summer, her aunt who lives in Calgary visits every third summer, and her aunt who lives in Toronto visits every second summer. The family had a reunion when Sarah was 6 years old. Sarah's dad is planning another reunion when all his sisters visit again. How old will Sarah be at the next reunion? **Interview** 1. Ask students why the GCF has to be a factor of the LCM. **Journal Entry** 1. Explain how you would find the LCM for 4, 7 and 5. **Performance** 1. Have students find LCMs using multiple strips.

STRAND: NUMBER (NUMBER CONCEPTS)

GENERAL OUTCOME

Demonstrate a knowledge of the interrelationship of the sets of numbers within the real number system.

SPECIFIC OUTCOME

3. Demonstrate and explain the meaning of proper and improper fractions. [C, R, V] (6–9)

MANIPULATIVES

- Fraction blocks
- Fraction strips
- Fraction circles

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Interactions 7*, p. 63
- *Mathpower 7*, pp. 48–49
- *Minds on Math 7*, pp. 92–102
- *TLE 7*, Fraction Conversions, Student Refresher pp. 10–11, Teacher's Manual pp. 32–35 (review only)

Previously Authorized Resources
- *Journeys in Math 8,* pp. 142–144
- *Journeys in Math 9*, pp. 20–21

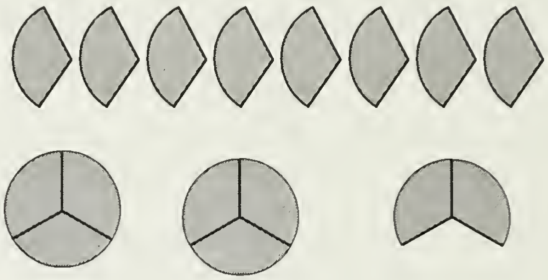
TECHNOLOGY CONNECTIONS

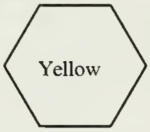
	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes It is important that students understand the concepts associated with fractions before they develop calculation skills using algorithms. Otherwise students don't understand why the algorithms apply. When students participate in activities using symbols, they should be able to use manipulatives to confirm their answers. Visualization is important in developing the meaning of fractions. Building patterns from blocks can be very effective.	This outcome should not require a great deal of time. Use common everyday situations, such as the following, as the focus for informal class discussion to determine what students already know about fractions. You and a friend ordered two pizzas, each cut into 6 equal slices. You eat all of your pizza and one slice from your friend's pizza. Express the amount of pizza you ate as an improper fraction. If your friend ate the remaining pizza, express the amount she ate as a proper fraction.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

1. Use fraction circles to a show that $\frac{8}{3}$ is equivalent to $2\frac{2}{3}$.

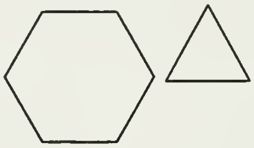


2. Using pattern blocks, let 

represent 1 whole.

Then, if 

represents $\frac{1}{6}$, illustrate $1\frac{1}{6}$ as a mixed number and as an improper fraction.



$1\frac{1}{6}$

$\frac{7}{6}$

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

In order to relate improper fractions to a number line, the relationship between improper fractions and mixed numbers should be developed.

Paper and Pencil

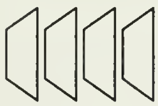
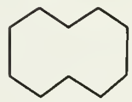
1. The complete rectangle represents one whole. For each diagram, what fraction is represented by the shaded portions?

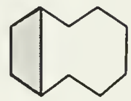
   a. 

   b. 

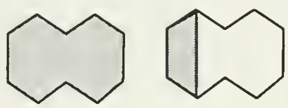
   c. 

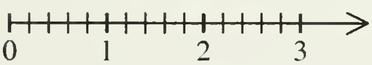
   d. 

2. If  are needed to build , then:

   a. What fraction of the whole does the shaded portion represent? 

   b. What fraction of the whole does the shaded portion below represent? (answer in improper fraction form)

3. 

   Show the position that represents:

   a. $\frac{3}{5}$

   b. $\frac{12}{5}$

4. The sketch to the right shows the pattern for a stained glass project.

   a. If the shaded portion of the pattern uses $\frac{1}{3}$ of a pane of coloured glass, how many panes of glass will be used in completing this design?

Performance

1. Use pattern blocks, where the value of the yellow hexagon is one whole, to make a pattern with a value of $2\frac{2}{3}$.

STRAND: NUMBER (NUMBER CONCEPTS)

GENERAL OUTCOME

Demonstrate a knowledge of the interrelationship of the sets of numbers within the real number system.

SPECIFIC OUTCOME

4. Distinguish between exact values and decimal approximations of square roots and cube roots. [E, T] (8–8)

MANIPULATIVES

- Square tiles
- Grid paper

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Addison-Wesley Mathematics 10*, pp. 18–20
- *Interactions 8*, pp. 42–52
- *Interactions 9*, pp. 43–44, 84–85
- *Mathpower 8*, pp. 28–31
- *Mathpower 9*, pp. 8–11
- *Minds on Math 9*, pp. 300–304
- *TLE 8*, Square Roots, Student Refresher pp. 10–11, Teacher's Manual pp. 32–35
- *TLE 9*, Square Roots, Student Refresher pp. 6–7, Teacher's Manual pp. 24–27
- *TLE 10*, Approximating Irrational Numbers, Student Refresher pp. 8–9, Teacher's Manual pp. 28–31 (the only place for cube roots)

Previously Authorized Resources
- *Journeys in Math 8*, pp. 18–19
- *Journeys in Math 9*, pp. 106–110

TECHNOLOGY CONNECTIONS

- Scientific calculator

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes Students should be aware of the perfect squares from 1 to 625. They should know the common squares and at least recognize that a number is a perfect "square." For example, students should know $25^2 = 625$ and should recognize 289 as being a perfect square. Some time should be spent on this before beginning the outcome. This is used again in working with the Pythagorean theorem.	A simple introduction to this activity is to have students compare their calculator answers to the square roots of perfect squares and of non-perfect squares. Some will round at a different number of digits and show that these are approximations. Another option is to have students write down the calculator display for $\sqrt{2}$ (1.414 …), clear the calculator, and then use the calculator to multiply that number by itself. This sometimes can demonstrate that the decimal value is not exact.

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

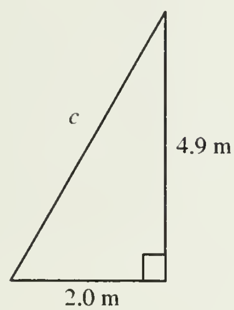
Teaching Notes

1. A player at third base must throw the ball to home plate. If the area of a square baseball diamond is 751 m^2, what is the distance from third base to home plate?

   Since
   $$A = S^2$$
   $$751 = S^2$$
   $$\sqrt{751} = S$$
   $$27.4 \doteq S$$

   The approximate distance is 27.4 m, which is $\sqrt{751}$ m rounded to the nearest tenth. The exact distance is $\sqrt{751}$ m.

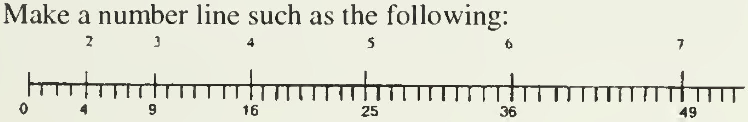
2. A power pole that is 4.9 m high is held up by a guy wire anchored 2.0 m from its base. How long is the guy wire?



$$a^2 + b^2 = c^2$$
$$4.9^2 + 2.0^2 = c^2$$
$$28.01 = c^2$$
$$\sqrt{28.01} = c$$
$$5.3 \doteq c$$

   The guy wire is approximately 5.3 m long. The exact measure is $\sqrt{28.01}$ m.

3. Have students use known squares to estimate square roots; e.g.: Make a number line such as the following:



   To estimate the square root of 28, find it on the number line and decide where it is in relation to the nearest squares and corresponding square roots. For example, $\sqrt{28}$ is less than 5.5 and is approximately equal to 5.3.

4. Have students use known cubes to estimate cube roots.

Number	1	2	3	4
Cube	1	8	27	64

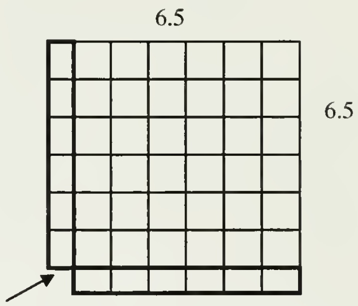
   The cube root of 35 is between 3 and 4 since 35 lies between 27 and 64.

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	The following activity may be used to approximate the square root of a number. However, it requires a considerable amount of time.

1. Using grid paper (1 cm × 1 cm), mark out an area that is 6 squares by 6 squares. Cut an additional strip that is 1 cm × 6 cm. How many squares do you have in all? (42)

   Cut the 1 cm × 6 cm strip in half lengthwise, and align these two strips on adjacent sides of your original square. Use the result to estimate $\sqrt{42}$ (Diagram 1).

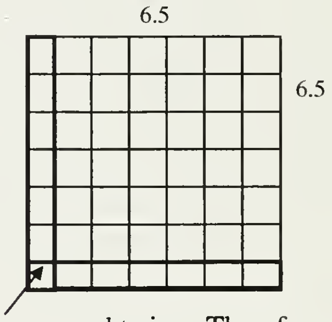
   Diagram 1

   This does not quite make a complete square, so $\sqrt{42} \cong 6.5$ but is slightly less than 6.5.

   Use a similar procedure to estimate $\sqrt{43}$ (Diagram 2).

   Diagram 2

   This square is covered twice. Therefore, $\sqrt{43}$ is slightly more than 6.5.

STRAND: NUMBER (NUMBER CONCEPTS)

GENERAL OUTCOME

Demonstrate a knowledge of the interrelationship of the sets of numbers within the real number system.

SPECIFIC OUTCOME

5. Differentiate between principal square root and positive and negative square roots of a number. Give examples where answers would involve the positive (principal) square root, or both positive and negative square roots of a number. [C, CN, PS, R] (9–3)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Addison-Wesley Mathematics 10*, pp. 18–20, 25
- *Interactions 9*, pp. 84–85
- *Mathpower 8*, pp. 28–31
- *Mathpower 9*, pp. 8–11
- *Minds on Math 9*, p. 306
- *TLE 8*, Square Roots, Student Refresher pp. 10–11, Teacher's Manual pp. 32–35

Previously Authorized Resources

- *Journeys in Math 8*, p. 18
- *Journeys in Math 9*, pp. 106–107
- *Math Matters: Book 2*, pp. 50–53

TECHNOLOGY CONNECTIONS

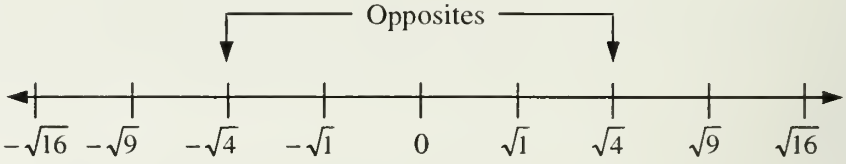
- Scientific calculator

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes Recognize that $-\sqrt{4}$ is different from $\sqrt{-4}$. $-\sqrt{4}$ is $^{-}2$, but $\sqrt{-4}$ is not a real number.	Just as subtraction is the opposite of addition and division is the opposite of multiplication, the opposite of squaring a number is to find the square root. For example, $2^2 = 2 \times 2 = 4$ and $(^{-}2)^2 = (^{-}2) \times (^{-}2) = 4$. Therefore, the square roots of 4 are 2 and $^{-}2$. Positive numbers always have two square roots: one positive and the other negative. The symbol $\sqrt{4}$ stands for the **positive** square root of 4. The positive square root is also called the **principal** square root. Thus $\sqrt{4} = 2$ The symbol $-\sqrt{4}$ stands for the **negative** square root of 4. $-\sqrt{4} = {}^{-}2$

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS

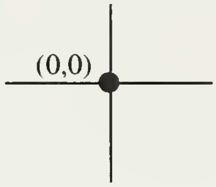
Teaching Notes

From the following number line, $\sqrt{4}$ and $-\sqrt{4}$ are opposites.



1. If you want to find the length of one side of a square garden whose area is 25 m^2, explain why you would use only the positive square root of 25.

2. A square has one vertex at (0, 0) and an area of 25 square units. Find the coordinates of the other vertices for four such squares.



The extension requires Pythagorean theorem.

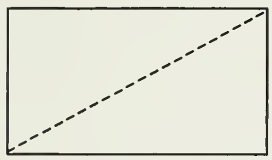
Extension: There are more squares than the four obvious ones. Find a few of them, and explain how they are found.

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

1. In each of the following situations, determine if both positive and negative square roots are appropriate or if just the principal square root is appropriate.

   a. Tracy was asked to find the length of the diagonal of this rectangle. Should she use both square roots, or only the principal square root?

   12 cm

   18 cm

$$
\begin{aligned}
a^2 + b^2 &= c^2 \\
12^2 + 18^2 &= c^2 \\
144 + 324 &= c^2 \\
468 &= c^2 \\
\sqrt{468} &= c
\end{aligned}
$$

   b. Travis was asked to solve the following equation. His work is shown. Should he use both square roots, or only the principal square root?

$$
\begin{aligned}
2x^2 + 17 &= 67 \\
2x^2 &= 50 \\
x^2 &= 25 \\
x &= \sqrt{25}
\end{aligned}
$$

   c. The result when 7 is taken away from the square of a number is 18. What are the possible values for the number?

STRAND: NUMBER (NUMBER CONCEPTS)

GENERAL OUTCOME

Develop a number sense of powers with integral exponents and rational bases.

SPECIFIC OUTCOME

6. Recognize and illustrate the meaning of a power, base, coefficient and exponent, including rational numbers or variables as bases or coefficients. [R, V] (9–4)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Addison-Wesley Mathematics 10*, pp. 52–55
- *Interactions 7*, pp. 38–39
- *Interactions 8*, pp. 35–40
- *Mathpower 7*, pp. 18–19
- *Mathpower 8*, pp. 4–5
- *Mathpower 9*, pp. 16–18, 26–29
- *Minds on Math 7,* pp. 30–31
- *Minds on Math 8,* pp. 456–461
- *Minds on Math 9,* pp. 276–279, 292–295
- *TLE 7*, Powers and Exponents, Student Refresher pp. 2–3, Teacher's Manual pp. 16–19
- *TLE 9*, Powers, Bases and Exponents, Student Refresher pp. 8–9, Teacher's Manual pp. 28–31
- *TLE 9*, Laws of Exponents 1–2, Student Refresher pp. 10–13, Teacher's Manual pp. 32–39

Previously Authorized Resources

- *Journeys in Math 8*, pp. 16–17
- *Journeys in Math 9*, pp. 94–95

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	**Exponents and Powers** In order to develop skills with exponents, students have to know the names of these parts. 4 is called the **exponent.** It tells you to multiply 4 factors of 2. 2^4 2^4 is called the exponent form. 2 is called the **base**. It tells you what number is multiplied. 2^4 is called a power. Read: 2 to the exponent 4 or the fourth power of 2 $2^4 = 2 \times 2 \times 2 \times 2$ (expanded form) Some powers have common names. $5^2 \leftarrow$ 5 squared $4^3 \leftarrow$ 4 cubed

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	The numerical factor of a term is called the coefficient. In the expression $2x^4$, the 2 is considered to be the coefficient of the power x^4. The coefficient of the term $-x^2$ is -1.

Term	Coefficient
$2x^4$	2
$3m^2n$	3
$-5y$	-5
$-x$	-1
x	1
$\frac{x^2}{3}$	$\frac{1}{3}$

Examples

1. What is the value of the coefficient in each of the following expressions?
   a. $-x^4$ b. $\frac{x^2}{5}$

Simplify Rational Numbers with Whole Numbers as Exponents

Students need to evaluate powers using pencil and paper.

Examples

1. Evaluate.
   a. 3^2 b. $^-3^2$ c. $(^-3)^2$ d. $\left(\frac{2}{3}\right)^3$ e. $(0.03)^2$ f. $\frac{2^3}{3}$
2. Which is larger, 3^4 or 4^3?

Some older scientific calculators cannot find the power if the base is negative. Most newer calculators, and all graphing calculators, can handle negative bases.

3. Students need to be able to evaluate powers with calculators. Evaluate:
   a. 2.7^8
   b. 389^3
   c. 2^{31}
   d. $^-3^4$
   e. $(^-3)^2$
4. Which of the following symbols, <, > or = would you place between $^-5^2$ and $(^-5)^2$ to make a true statement?

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	5. Evaluate these powers mentally. a. $\left(\frac{2}{5}\right)^2$ b. $^{-}2^4$ c. $(0.06)^2$ d. What is the last digit of 5^8? **Evaluate Algebraic Expressions with Whole Number Exponents** Remind students to replace each variable with a set of parentheses **and then** place the number inside the parentheses. If $x = {}^{-}3$, evaluate $^{-}x^2$. $^{-}x^2 = -({}^{-}3)^2$ $= -(9)$ $= {}^{-}9$ Examples 1. If $x = {}^{-}2$, find the value of x^2. Answer $x^2 = ({}^{-}2)^2$ $= 4$ 2. If $x = {}^{-}5$ and $y = {}^{-}3$, find the value of $^{-}2xy^2$. Answer $^{-}2xy^2 = {}^{-}2({}^{-}5)({}^{-}3)^2$ $= {}^{-}2({}^{-}5)(9)$ $= 90$ 3. If $x = 2$ and $y = 3$, find the value of $x^y - y^x$. Answer $x^y - y^x = (2)^3 - (3)^2$ $= 8 - 9$ $= {}^{-}1$ 4. If x represents an integer, which integers satisfy: a. $x^2 = 1$ b. $x^2 < 1$ c. $x^2 > 1$ 5. Arrange x^2, x^3 and $^{-}x^3$ in decreasing order, if $x = -\frac{1}{2}$.

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Paper and Pencil** 1. Find the value of $(x + y)(x^2 - xy + y^2)$, if $x = {}^-1$ and $y = 2$. 2. Express the number 48 in exponent form. 3. Which is larger, 5^3 or 3^5? **Performance** 1. John wants to use his calculator to find 9^4, but the 4 key is missing. a. Explain how he can use the calculator to find the answer to this question, even though the 4 is missing. b. Suppose the 9 key is missing instead. Explain how he might now use the calculator to find the answer. **Interview** 1. Explain to students that Susan was asked to find the volume of a cube that was 4 cm on one side. She wrote: 4^3 as $3 \times 3 \times 3 \times 3 = 81$. Ask students if the solution is correct and also to explain why or why not. **Portfolio** 1. Ask students to find a value for $\square$ and a value for * which would make the following sentence true: $3^\square = 9^*$. Ask if there are other values for $\square$ and * that would work. 2. Use patterning to help you find the **last** digit in: a. 4^{100} b. $({}^-2)^{101}$ c. 5^{50} Why can't you use your calculator to answer this? **Journal** 1. Explain the difference between 2^3 and 3^2.

STRAND: NUMBER (NUMBER CONCEPTS)

GENERAL OUTCOME

Develop a number sense of powers with integral exponents and rational bases.

SPECIFIC OUTCOME

7. Explain and apply the exponent laws for powers with integral exponents.

$x^m \bullet x^n = x^{m+n}$

$x^m \div x^n = x^{m-n}$

$(x^m)^n = x^{mn}$

$(xy)^m = x^m y^m$

$\left(\frac{x}{y}\right)^n = \frac{x^n}{y^n}, y \neq 0$

$x^0 = 1, x \neq 0$

$x^{-n} = \frac{1}{x^n}, x \neq 0$

[PS, R] (9–5)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Addison-Wesley Mathematics 10*, pp. 52–58
- *Interactions 9*, pp. 28–42
- *Mathpower 9*, pp. 20–21, 24–43
- *Minds on Math 9*, pp. 276–299
- *TLE 9*, Laws of Exponents 1–3, Student Refresher pp. 10–15, Teacher's Manual pp. 32–43
- *TLE 9*, Evaluating Powers and Expressions, Student Refresher pp. 16–17, Teacher's Manual pp. 44–47

Previously Authorized Resources

- *Journeys in Math 9*, pp. 96–101
- *Math Matters: Book 2*, pp. 95–100

TECHNOLOGY CONNECTIONS

- Scientific calculator

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	The exponent laws, using variable bases, are only touched on briefly in the junior high school curriculum. The concentration is on integral bases. As with numerical bases, it is important to develop the laws by using the expansion of powers, then simplifying to illustrate each law; e.g.:

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS[1]

Teaching Notes

a. $x^3 \cdot x^2 = (x \cdot x \cdot x) \cdot (x \cdot x) = x^5$

b. $\dfrac{x^5}{x^3} = \dfrac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x}} = x^2$

c. $(x^3)^2 = (x^3)(x^3) = (x \cdot x \cdot x)(x \cdot x \cdot x) = x^6$

d. $(xy)^3 = (x \cdot y)(x \cdot y)(x \cdot y) = (x \cdot x \cdot x)(y \cdot y \cdot y) = x^3 y^3$

e. $\left(\dfrac{x}{y}\right)^3 = \left(\dfrac{x}{y}\right)\left(\dfrac{x}{y}\right)\left(\dfrac{x}{y}\right) = \dfrac{x^3}{y^3}$

f. $$\frac{x^3}{x^3} \begin{cases} = x^{3-3} = x^0 \\ = \dfrac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x}} = 1 \end{cases}\Bigg\}\ \text{therefore, } x^0 = 1$$

g. $$\frac{x^3}{x^5} \begin{cases} = x^{3-5} = x^{-2} \\ = \dfrac{\cancel{x} \cdot \cancel{x} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{x} \cdot \cancel{x} \cdot x \cdot x} = \dfrac{1}{x^2} \end{cases}\Bigg\}\ \text{therefore, } x^{-2} = \frac{1}{x^2}$$

1. Write equivalent expressions with positive exponents.

   a. $\dfrac{x^{-2}}{y}$

   b. $\dfrac{x^2}{y^{-3}}$

   c. $\dfrac{x^{-2}y}{z^{-6}}$

   d. $\dfrac{-2x^{-2}}{y^{-4}}$

2. Express $2^{-1} + 4^{-1}$ as the sum of two rational numbers.

[1] Instructional Strategies/Suggestions questions 1 to 3 are reproduced, by permission, from Manitoba Education and Training. *Senior 1 Mathematics: A Foundation for Implementation*. Winnipeg, MB: Manitoba Education and Training, 1997.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	3. Using a variety of operations, write expressions that are equivalent to x^2. Use your imagination. 4. Evaluate: a. 3^{-2} b. -3^{-2} c. $(-3)^{-2}$ d. $\frac{2^{-2}}{3}$ e. $\frac{1}{2^{-3}}$ 5. Simplify, removing negative exponents and parentheses. a. $\left(\frac{2}{3}\right)^{-5}$ b. $(35^2)^3$ c. $(xy)^m$ d. $\left(\frac{x}{y}\right)^n$ e. $\left(\frac{x}{y}\right)^{-n}$ 6. The quantities in bold type may contain errors. By correctly applying the exponent laws, replace incorrect answers with correct answers. a. $m^4 \bullet m^3 = \boldsymbol{m^{12}}$ b. $m^{12} \div m^2 = \boldsymbol{m^6}$ c. $(m^5)^2 = \boldsymbol{m^7}$ d. $(xy)^6 = \boldsymbol{x^1}\ y^6$ e. $\left(\frac{x}{y}\right)^3 = \left(\frac{\boldsymbol{x^3}}{\boldsymbol{y^1}}\right)$ f. $\left(\frac{2^3}{3}\right) = \left(\frac{\boldsymbol{2^3}}{\boldsymbol{3^3}}\right)$ g. $3^0 = \mathbf{0}$ h. $5^1 = \mathbf{1}$ 7. Explain why $5^0 = 1$, not zero.

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	See below

Paper and Pencil

1. Predict and verify to find values for x.

   a. $x^4 \times x^2 = 64$
   b. $x^8 \div x^3 = 243$
   c. $(x^2)^4 = 256$
   d. $(x^3)^{-2} = 64$

2. Provide the missing exponents.

   a. $10^{-5} \div 10^5 = 10^{\square}$
   b. $10^{-6} \div 10^{\square} = 10^{-14}$
   c. $\dfrac{10^{12}}{10^{-5}} = 10^{\square}$
   d. $(-5)^{15} = [(-5)^{\square}]^3$
   e. $3^5 \times 4^3 = (3 \times 4)^{\square} = 12^{\square}$
   f. $5 \div (-6)^{-7} = 5^{\square} \times (-6)^{\square}$

Journal

1. Write in your own words what is meant by a power of a power.

2. Copy and complete the following power chart. Describe any patterns you see. Make a similar chart using 3 as the base; compare the charts, and describe their similarities.

$$
\begin{aligned}
2^5 &= 32 \\
2^4 &= 16 \\
2^3 &= \\
2^2 &= \\
2^1 &= \\
2^0 &= \\
2^{-1} &= \frac{1}{2} \\
2^{-2} &= \frac{1}{4} \\
2^{-3} &= \frac{1}{8} \\
2^{-4} &= \\
2^{-5} &=
\end{aligned}
$$

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Interview**❶ 1. Ask students to explain two ways that $(2 \times 5)^4$ can be calculated. 2. $2^4 \times 2^{-2} \times 5^3 \times 5^{-3} \times 10^5 \times 10^{-4}$ a. Ask students to explain why this is easy to solve mentally. b. Ask students to solve the problem mentally. c. Ask students to write a similar problem involving six powers that is also easy to solve mentally. Have the students exchange their problems with fellow students.

❶ Interview questions 1 and 2 are adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 9 Implementation Draft*.

STRAND: NUMBER (NUMBER OPERATIONS)

GENERAL OUTCOME

Use a scientific calculator to solve problems involving real numbers.

SPECIFIC OUTCOME

8. Document and explain the calculator keying sequences used to perform:
   - square roots, cube roots
   - exponents
   - scientific notation
   - sine, cosine, tangent
   - integers.

   [PS, R, T] (9–10)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *TLE 9*, Evaluating Powers and Expressions, Student Refresher pp. 16–19, Teacher's Manual pp. 44–51
- *TLE 9*, Ratios in Right Triangles, Student Refresher pp. 58–59, Teacher's Manual pp. 128–131
- *TLE 9*, Finding Unknown Angles, Student Refresher pp. 62–63, Teacher's Manual pp. 136–139

Previously Authorized Resources
- *Math Matters: Book 2*, pp. 31–53
- *Mathematics 9,* pp. 7, 24, 46, 57, 146, 162, 163, 258, 342

TECHNOLOGY CONNECTIONS

- Scientific calculator

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	This specific outcome can be approached in the appropriate context throughout the course. Students should be able to communicate keying instructions to perform these operations. Since different calculators may require different keying sequences, you may choose to develop the most common sequence(s) as a group. Individual calculator differences may be dealt with separately. Within the context of integer work, the order of operations usually requires appropriate use of parentheses.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Example 1. To find the square root of 2, you may input one of these possibilities: a. $\boxed{\sqrt{\ }}$ 2 b. 2 $\boxed{\sqrt{\ }}$ c. $\boxed{2^{nd}}$ $\sqrt{\boxed{x^2}}$ 2 d. 2 $\boxed{2^{nd}}$ $\sqrt{\boxed{x^2}}$ Each student should be expected to communicate the keystrokes for the calculator he/she uses. In this example, a key is labelled $\boxed{2^{nd}}$. Alternative labels include $\boxed{\text{shift}}$ or $\boxed{\text{INV}}$. Also, students should be reminded to put their calculators in DEGREE mode before finding sine, cosine or tangent.

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Journal**

Journal

1. Students may keep a section of their notebook to record keystroke sequences for each operation as that operation is developed throughout the course. The keystroke sequence would depend on the calculator that the student uses and the solution strategy used.

Paper and Pencil (calculator)

1. a. Multiply 1 600 000 × 4 000 000.
   How does your calculator show the answer?
   If your calculator displays $\boxed{6.4 \quad 12}$, what is your answer in scientific notation?

   b. Multiply 0.000 053 × 0.000 000 3.
   If your calculator displays $\boxed{1.59 \quad -11}$, what is your answer in scientific notation?

2. a. Estimate $(1.9 \times 3.1)^3$

   b. Calculate $(1.9 \times 3.1)^3$, using the keying sequence

   $\boxed{(}$ 1.9 $\boxed{\times}$ 3.1 $\boxed{)}$ $\boxed{y^x}$ 3 $\boxed{=}$

   c. If you calculate without the brackets, do you get the same answer? Explain.

   d. Calculate $(-2)^6$ and -2^6. Compare your answers and explain.

   e. Mentally calculate $\frac{4^2}{2+6}$. Write a keying sequence you could use if you used your calculator.

These keying sequences are not unique, even for the same calculator.

Interview

1. Two students or a student and the teacher exchange calculators. One explains the keying sequence for solving a problem to the other. Then their roles are reversed.

STRAND: NUMBER (NUMBER OPERATIONS)

GENERAL OUTCOMES

Demonstrate an understanding of and proficiency with calculations on rational numbers.

Decide which arithmetic operations can be used to solve problems and then solve the problem.

SPECIFIC OUTCOME

9. Perform arithmetic operations with integers concretely, pictorially and symbolically. [PS, V] (7–16)

MANIPULATIVES

- Algebra tiles
- Two-sided counters
- Bingo chips
- Number lines
- Thermometers
- Coloured squares

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Interactions 7*, pp. 94–102, 276–287
- *Mathpower 7*, pp. 116–135
- *Minds on Math 7*, pp. 270–304
- *Minds on Math 8*, pp. 270–275
- *Minds on Math 9*, pp. 26–31
- *TLE 7*, Exploring Integers, Student Refresher pp. 28–29, Teacher's Manual pp. 68–71
- *TLE 7*, Adding Integers, Student Refresher pp. 30–31, Teacher's Manual pp. 72–75
- *TLE 7*, Subtracting Integers, Student Refresher pp. 32–33, Teacher's Manual pp. 76–79

Previously Authorized Resources
- *Journeys in Math 8*, pp. 280–291
- *Journeys in Math 9*, pp. 38–51

TECHNOLOGY CONNECTIONS

- Scientific calculator
- Spreadsheet

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Manipulatives and concrete models should be used to develop the concept of calculations with integers. Useful manipulatives include algebra tiles, bingo chips, two-sided counters, number lines, thermometers and coloured squares. Results should then be connected to symbolic representations. When using algebra tiles (two coloured items) the zero principle is important; i.e., different colours cancel each other.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

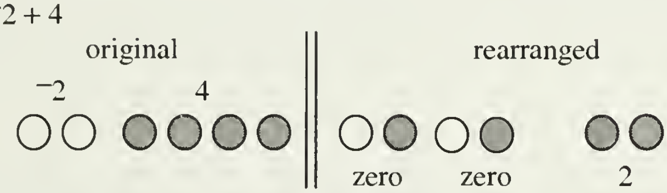
Using algebra tiles, express the number 3 in as many ways as possible; e.g.:

Ask students to represent:

a. $^{-}4$, using 6 coloured counters
b. zero, using 6 coloured counters
c. +2, using 6 coloured counters

Adding Integers

Using Tiles: When adding two integers, it is necessary to first model each integer, then match positive and negative values to make zeros; e.g., $^{-}2 + 4$



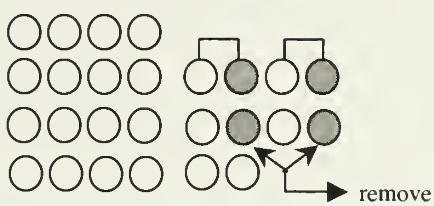
Therefore, $^{-}2 + 4 = 2$ (symbolic connection).

Subtracting Integers[1]

If you don't want to use the regrouping method, you will need to use double-sided tiles.

Using Tiles: For subtraction, Elaine owes \$4 and she borrows \$2 more from a friend. This may be represented as follows, using the zero principle.

1. Begin with $^{-}4$.
2. Add extra zeros.
   This is still $^{-}4$.
3. Remove +2
4. Answer
   $^{-}4 - 2 = {}^{-}6$



Have students work in pairs. Ask them each to roll two dice of different colours. Assign negative to one colour and positive to the other, and write a number sentence for the sum. Have them roll the two dice again, find the sum mentally and add the result to their previous score. Have them exchange turns until one person reaches +20 or $^{-}20$. Ask why it would be fair to accept +20 or $^{-}20$ as the winning score.

This activity can be modified and used with other operations. It can also be modified by assigning the negative or positive to specific colours after each roll, instead of maintaining the same designation throughout the game. Ask students if this change would allow them to get to +20 or $^{-}20$ more quickly. Have them consider other possible rule changes, such as "You can't go over 20."

[1] Information on Subtracting Integers is adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 7.*

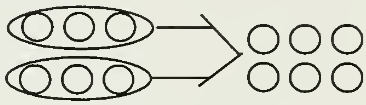
	INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

A similar game can be played using a deck of cards, where red represents one sign and black the other sign.

Multiplying Integers[1]

Use tiles to show groupings; e.g.:

2 (– 3) 

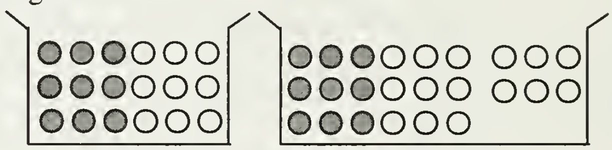
2 groups of $^{-}3$ gives $^{-}6$

Answer: $(2)(-3) = {}^{-}6$

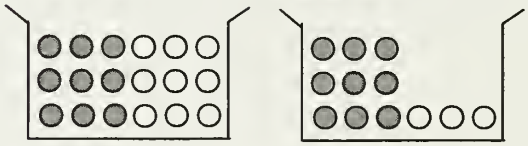
Addition of integers helps to establish some of the initial groundwork for multiplication of integers. Multiplication of integers should start with examining multiplication as repeated addition, as in:

4 sets of $({}^{-}3) = ({}^{-}3) + ({}^{-}3) + ({}^{-}3) + ({}^{-}3)$.

The following is one way of using counters to model multiplication. Start with a container having an equal number of positives and negatives.



+2 ($^{-}3$) implies adding 2 sets of $^{-}3$. What is the total in the container?



$^{-}2$ ($^{-}3$) implies removing 2 sets of $^{-}3$. When 2 sets of $^{-}3$ are removed, what remains in the container?

When the first factor of the multiplication is positive, the operation is conceptualized as repeated addition. When the first factor of the multiplication is negative, the operation can be conceptualized as repeated subtraction.

To summarize in words, we say that we multiply the digits, and the sign of the answer can be determined as follows: same signs give a positive answer while different signs result in a negative answer.

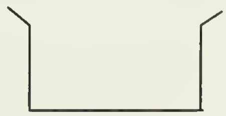
Dividing Integers[1]

Using Tiles: The following provides a starting point for modelling division.

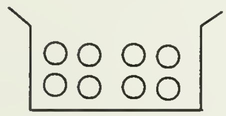
[1] Information on Multiplying Integers and Dividing Integers is adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 7.*

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	

Start with an empty container.



$^-8 \div (^-2)$
Ask students how many groups of $^-2$ are in $^-8$.



Add successive groups of $^-2$ to the container until there is $^-8$ in it. Record the number of groups. Four groups of $^-2$ were added to the container; therefore, $^-8 \div (^-2) = 4$.

The model can be used with some modification for other division situations.

$27 \div (^-9) =$

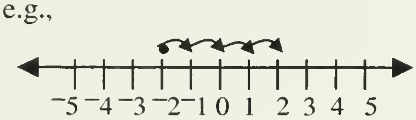
$^-60 \div (^-15) =$

A scientific calculator will also perform these operations when the proper keying sequences are used.

Adding Integers

Using a Number Line: Start at the first number. Adding a positive indicates movement to the right; adding a negative indicates movement to the left; e.g.,

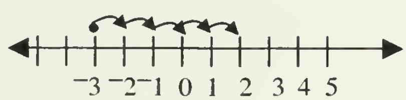
$^-2 + 4$



Therefore, $^-2 + 4 = 2$.

Subtracting Integers

Using a number line requires the knowledge that subtraction is the opposite of addition. For example $^-3 - (^-5) = {^-3} + (+5) = +2$



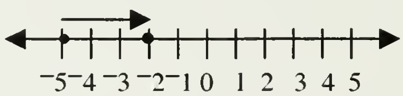
	INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

When using the number line to describe subtraction of integers, the use of the comparison model is more meaningful than the take-away model for subtraction. For example, $^-3 - (^-4)$ means how far is it from $^-4$ to $^-3$. The distance is 1. In going from $^-4$ to $^-3$, we move in a positive direction so the answer is +1. Note: When the second number of an addition or subtraction is negative, we use parentheses, such as: $^-4 + (^-3)$.

Subtraction may be easier for some students, by using a missing addend approach. For example, to find $^-8 - (^-4)$, ask: What would you add to $^-4$ to get $^-8$?

Show $^-2 - (^-5)$ on a number line.



Start with $^-5$.

Ask how far it is to $^-2$, and in what direction.

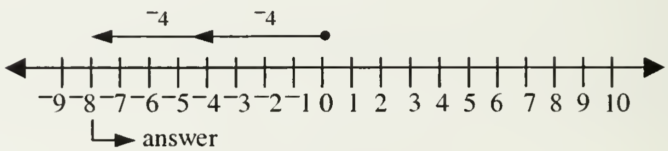
Answer is 3.

Multiplying Integers[1]

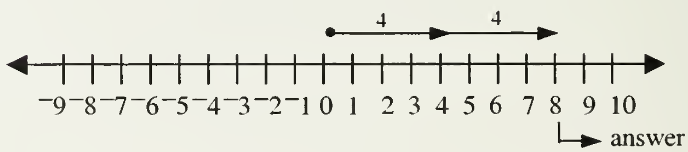
Using a Number Line: Net worth can also be used as a context for multiplication. Consider, for example, the impact on net worth if a person owes \$6 to each of three friends, or if a debt of \$6 to each of three friends is forgiven.

The number line can also be used to model problems such as:

$(2) \times (^-4) \rightarrow$ 2 sets of $^-4$



$(2) \times (+4) \rightarrow$ 2 sets of 4



[1] Information on Multiplying Integers is adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 7.*

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	

Patterning can then be used to justify the result for the multiplication of a negative by a negative.

$(3)(^-2) = {}^-6$

$(2)(^-2) = {}^-4$

$(1)(^-2) = {}^-2$

$(0)(^-2) = 0$

$(^-1)(^-2) = 2$

$(^-2)(^-2) = ?$

$(^-3)(^-2) = ?$

Dividing Integers[1]

Patterning is also useful in division; for example,

$6 \div 2 = 3$	${}^-6 \div (^-2) = 3$
$4 \div 2 = 2$	${}^-4 \div (^-2) = 2$
$2 \div 2 = 1$	$-2 \div (^-2) = 1$
$0 \div 2 = 0$	$0 \div (^-2) = 0$
${}^-2 \div 2 = ?$	$2 \div (^-2) = ?$
${}^-4 \div 2 = ?$	$4 \div (^-2) = ?$

Comparison of multiplication and division situations can also be very useful in helping students understand division of integers. After multiplication has been fully developed, the fact that multiplication and division are inverse operations can be utilized. For example, since ${}^-4 \times 3 = {}^-12$, it must be true that the product divided by either factor should equal the other factor; therefore, ${}^-12 \div (^-4) = 3$ and ${}^-12 \div 3 = {}^-4$. Likewise, if ${}^-4 \times (^-3) = 12$, then $12 \div (^-4) = {}^-3$ and $12 \div (^-3) = {}^-4$.

Using a missing factor can also be useful. For example, in the case of ${}^-16 \div (^-4)$, ask: what multiplied by ${}^-4$ gives ${}^-16$?

The concept of net worth can be linked with division as well. Students can think of owing \$12 when an equal amount is owed to each of three friends. Students can determine how much is owed to each friend.

[1] Information on Dividing Integers is reproduced with permission from *Atlantic Canada Mathematics Curriculum: Grade 7.*

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

Performance

1. Have students use a number line or coloured counters to explain:

   a. $^{-}3 + 8 = 5$
   b. $^{-}5 - 3 = {}^{-}8$
   c. $^{-}4 - ({}^{-}6) = 2$
   d. $9 + ({}^{-}2) = 7$
   e. $6 - 4 = 2$
   f. $8 - ({}^{-}3) = 11$

2. Fill in the grid of numbers below, and comment on the patterns you observe.

×	4	2	$^{-}2$	$^{-}4$	$^{-}6$
4	16				
2		4			
$^{-}2$					
$^{-}4$		$^{-}8$			
$^{-}6$					

Portfolio

1. Using a newspaper or the Internet, refer to worldwide resort temperatures. Record the highest and lowest temperature, and determine the difference.

2. Make a monthly budget for yourself. What items would be indicated by positive integers? Which would be indicated by negative integers?

Paper and Pencil ❶

1. Complete the following patterns:

$9 \div 3 = 3$	$^{-}9 \div ({}^{-}3) = 3$
$6 \div 3 = 2$	$^{-}6 \div ({}^{-}3) = 2$
$3 \div 3 = 1$	$^{-}3 \div ({}^{-}3) = 1$
$0 \div 3 = 0$	$0 \div ({}^{-}3) = 0$
___ $\div 3 = {}^{-}1$	___ $\div ({}^{-}3) = {}^{-}1$
___ $\div 3 =$ ___	___ $\div$ ___ $=$ ___
___ $\div$ ___ $=$ ___	___ $\div$ ___ $=$ ___
	___ $\div$ ___ $=$ ___

❶ Paper and Pencil questions 1 to 6 and 8 are reproduced with permission from *Atlantic Canada Mathematics Curriculum: Grade 7.*

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	

2. Based on the patterns, write a conclusion that can be drawn about division of a positive integer by a positive integer, a negative by a negative, a positive by a negative, and a negative by a positive. Confirm your conclusion by creating similar patterns beginning with

   a. $20 \div 4$
      $16 \div 4$

   b. $^-20 \div (^-4)$
      $^-16 \div (^-4)$

3. Write a number sentence to model the following situation. Chris and his three friends together owe $32. They agreed to share the debt equally. What is each person's share of the debt?

4. Write a division or multiplication sentence to solve each problem.
   a. $? \div (^-3) = {^-9}$
   b. $57 \div ? = {^-3}$

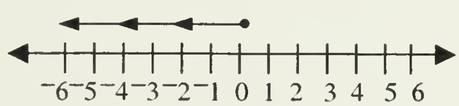
5. Write a number sentence for each of the following problems and use a diagram to model the situations.
   a. Fran lost 3 points in each round (hand) of cards that was played. If she played 4 rounds (hands), what was her score at the end of the game?
   b. Bill owed $5 to each of 3 friends. What was his net worth, based on this situation?

6. Tell students that a list of temperatures for seven days was prepared, but the ink became wet and one temperature was unreadable. The mean temperature was $^-3$°C, and the six known temperatures were 2°C, $^-4$°C, $^-6$°C, $^-2$°C, 4°C, and $^-5$°C. Ask students to find the missing temperature.

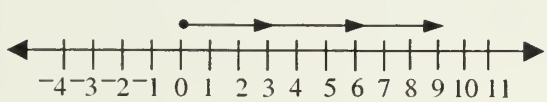
7. John saved $50 during the fall. He owes $15 to his friend. Because he had a good term report, his father gave him $20. What is John's net worth?

8. Write number sentences for each of the following:

   a.



   b.



	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	9. Solve: $^{-}12 + 14 =$ $(^{-}3)(^{-}17) =$ $^{-}60 \div 3 =$ $42 - 70 =$ $^{-}16 - (^{-}19) =$ **Journal** 1. a. Explain how adding and subtracting are related. b. Explain how multiplying and dividing are related. 2. Ask students if the following statement is true or false. The sum of a negative number and a positive number is always negative. Explain why or why not. 3. Tell students that a friend missed class the day that division of integers was first introduced. Ask them to write a detailed explanation for the friend to help him/her understand how to solve: a. $^{-}10 \div 5$ b. $^{-}24 \div (^{-}6)$ **Interview** 1. Ask students to name as many pairs of integers as possible that have a product of $^{-}16$ and then a product of $^{+}16$. Ask what they notice about the number of possible pairs for the positive product versus the negative product.

STRAND: NUMBER (NUMBER OPERATIONS)

GENERAL OUTCOMES

Demonstrate an understanding of and proficiency with calculations on rational numbers.

Decide which arithmetic operations can be used to solve problems and then solve the problem.

SPECIFIC OUTCOME

10. Illustrate and explain the order of operations. [PS, T, V] (7–17)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Interactions 7*, pp. 77–78, 286–287
- *Mathpower 7*, pp. 98–99
- *Mathpower 9*, p. 44
- *Minds on Math 7*, pp. 132–135, 300–304
- *Minds on Math 8*, pp. 469–472
- *TLE 7*, Decimal Tile Explorer
- *TLE 7*, Exploring Decimals, Student Refresher pp. 20–21, Teacher's Manual pp. 52–55
- *TLE 7*, Multiplication of Decimals, Student Refresher pp. 22–33, Teacher's Manual pp. 56–59
- *TLE 7*, Division of Decimals, Student Refresher pp. 24–25, Teacher's Manual pp. 60–63

Previously Authorized Resources

- *Journeys in Math 8*, pp. 22–23, 290–291
- *Journeys in Math 9*, pp. 12–13, 50–51, 74–75
- *Math Matters: Book 2*, pp. 6, 26

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes The most common error is $4 + 20 \div 2 \times 5 - 6 = 54$, where the students add the 4 + 20 first.	Introduce order of operations by giving an example, such as the following, where many answers can be derived. Example: Ask the students to find the answer $4 + 20 \div 2 \times 5 - 6$. Some typical answers could be $^{-}60$, 48, 0, $^{-}6$, 54. Ask the students to explain how they arrived at their answers. Since several answers can be logically explained, the students will see a need for order of operations. At this point, state the correct order in which operations are done with respect to the above example. • Do multiplication and division next, in the order in which they occur from left to right. • Do addition and subtraction last, in the order in which they occur from left to right.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Using the example and the various answers given at the beginning of the class, explain how each of the given answers could be correct if parentheses were inserted and the operations within the parentheses were done first. Students can then decide where to place the parentheses so that each answer could be correct; e.g.: $(4 + 20) \div 2 \times 5 - 6 = 54$ $4 + 20 \div (2 \times 5) - 6 = 0$ $4 + 20 \div 2 \times (5 - 6) = {}^{-}6$ Exponents and order of operations should be dealt with next.
Students have difficulty with determining the base.	Ask the students to explain the difference between the following three expressions: • ${}^{-}3 + ({}^{-}2)^2 \div 2$ • ${}^{-}3 + ({}^{-}2^2) \div 2$ • $({}^{-}3 + ({}^{-}2))^2 \div 2$ Order of Operations • Operations within brackets are performed first. • Operations with exponents are performed next. • Multiplication and division are performed next, in the order they occur from left to right. • Addition and subtraction are performed last, in the order they occur from left to right. When using the calculator, keying sequences should be checked. Find the result:
While some of these problems look complex, students need to break them down into smaller parts, solve the parts and calculate the answer. Estimates are essential before students use their calculators.	a. $[{}^{-}27 + 7({}^{-}3)] \div [({}^{-}3) \times 2^2]$ b. $51 \div [({}^{-}6)^2 - (3^2 + 10)] + 23 - [16 - (4 \times 3)]$ c. $(6 + 2) \div ({}^{-}2 + 4)^2 \times (25 - 5 + 6 - 10)$ d. $\dfrac{(-5)^2 - [3(-7)]}{(-2)^2 - (9 - 3)}$

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

Paper and Pencil

1. Solve:

   a. $6 + (^-3)(^-22)$
   b. $^-7 + 2 \times {}^-14$
   c. $[^-12 + {}^-3] \times (^-16) + {}^-4$

2. Using brackets, write the expression so that it equals the given number.

   a. $2 + 3^2 \times 7 - 4$ $\quad 33$
   b. $72 \div 4 + 2^2 + 1$ $\quad 8$
   c. $10 + 5 + 3^3 \div 6$ $\quad 7$
   d. $10 - 7 - 2^2 \times 3$ $\quad {}^-45$

3. Evaluate the following.
   a. $(17 - (^-4)) \times 2$
   b. $6^2 - 2^2 \div 4$
   c. $120 \div [18 - (^-1)^2 + 3]$
   d. $(^-3 + 4)(8 - 10) - (6 - 8)(7 - 5)$
   e. $(5 + 4) \div (8 - 7) + (16 + 4) \div (^-5)$
   f. $\dfrac{^-24 - 16 \times (^-2)}{6^2 \div 9}$
   g. $\dfrac{(^-6)(^-3) + (^-3)(4)}{(^-2)^2 + 8 \div 4}$

Portfolio

1. Collect contest skill-testing questions and solve them.

2. Use the digit 3 exactly 4 times separated by operations signs and/or parentheses to generate the numbers from 0 to 10; e.g.:
   $3 \times 3 - 3 \times 3 = 0$
   $(3 + 3 + 3) \div 3 = 3$

Performance[1]

1. Ask students to write a number sentence for the following and solve it, using the order of operations.
   a. Ms Jones bought the following for her project: 5 sheets of pressboard at $8.95 a sheet, 20 planks at $2.95 each, and 2 litres of paint at $9.95. What was the total cost?

[1] Performance questions 1 to 6 are adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 7.*

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	b. Three times the sum of $34.95 and $48.95 represents the total amount of Jim's sales on April 29. When his expenses, which totalled $75.00, were subtracted, what was his profit? c. Consider solving the number sentences for parts a and b by ignoring the order of operations. Would the solutions make sense in terms of the problems? Discuss. 2. Why is there an order of operations? Terry and Pat both answered a skill-testing question: $10 + 3 \times 2 - 1$. Terry got 15 and Pat got 25. Who was right and why? 3. Ask students to explain why it is necessary to know the order of operations to compute $4 \times 7 - 3 \times 6$. Ask them to compare the solution of this problem with the solution of $4 \times (7 - 3) \times 6$. Ask if the solutions are the same or different and why. 4. Owing to some faulty keys, the operation signs in these problems did not print. Use the information that is supplied to help determine which operators were used. a. $(7.4 \ \square\ 2) \ \square\ 12.6 = 2.2$ b. $2 \ \square\ 7 \ \square\ 2 \ \square\ 3 = 13$ 5. Because the shift key of the keyboard did not work, none of the parentheses appeared in these problems. If the student has the right answer to both problems, identify where the parentheses must have been. a. $4 + 6 \times 8 - 3 = 77$ b. $26 - 4 \times 4 - 2 = 12$ 6. Billy had to answer the following skill-testing questions to win the contest prize. What are the winning answers? a. $234 \times 3 - 512 \div (2 \times 4)^2$ b. $18 + 8 \times 2 - 32 \div 4$ Billy was told that the correct answer for part b is 5, but Billy disagreed. What did the contest organizers do in solving the question that caused them to get 5 for the answer? Explain why you think they made that error.

STRAND: NUMBER (NUMBER OPERATIONS)

GENERAL OUTCOMES

Demonstrate an understanding of and proficiency with calculations on rational numbers.

Decide which arithmetic operations can be used to solve problems and then solve the problem.

SPECIFIC OUTCOME

11. Add, subtract, multiply and divide fractions concretely, pictorially and symbolically. [E, PS, V] (8–9)

MANIPULATIVES

- Fraction circles
- Fraction strips

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Interactions 8*, pp. 88–123
- *Mathpower 8*, pp. 46–59
- *Minds on Math 8*, pp. 54–112
- *TLE 8*, Fractions Explorer
- *TLE 8*, Subtracting Fractions (unlike denominators), Student Refresher pp. 18–19, Teacher's Manual pp. 48–51
- *TLE 8*, Multiplying Fractions, Student Refresher pp. 20–21, Teacher's Manual pp. 52–55
- *TLE 8*, Dividing Fractions, Student Refresher pp. 22–23, Teacher's Manual pp. 56–59
- *TLE 8*, Exploring Fractions, Student Refresher pp. 12–13, Teacher's Manual pp. 36–39
- *TLE 8*, Adding and Subtracting Fractions, Student Refresher pp. 14–15, Teacher's Manual pp. 40–43
- *TLE 8*, Adding Fractions (unlike denominators), Student Refresher pp. 16–17, Teacher's Manual pp. 44–47

Previously Authorized Resources
- *Journeys in Math 8*, pp. 140–141, 146–151
- *Journeys in Math 9*, pp. 20–23

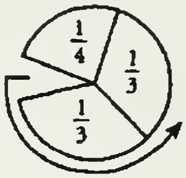
TECHNOLOGY CONNECTIONS

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

Students should be able to perform simple calculations of fractions **without the use of a calculator.**

1. A brief review of the following concepts may be necessary: equivalent fractions, lowest terms and LCM. A number of manipulatives can be used to develop operations with fractions concretely, including circle models, pattern blocks, tangrams, money, number lines, fraction factory pieces, fraction bars and other fraction kits. Students should model with fractions; e.g.:



$\frac{1}{4}+\frac{1}{3}+\frac{1}{3}=\frac{11}{12}$

Since $\frac{1}{12}$ fits the gap, $\frac{11}{12}$ must be the sum.

$\frac{5}{6}-\frac{1}{3}$

Start with $\frac{5}{6}$ 

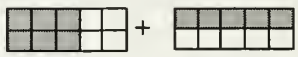
Represent $\frac{1}{3}$ 

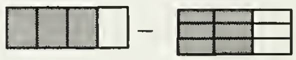
Place the $\frac{1}{3}$ piece over the $\frac{5}{6}$ and $\frac{3}{6}$ are left.

When moving toward developing an algorithm for addition and subtraction of fractions, a major focus should be placed on the writing of equivalent fractions. Once students internalize the fact that fractions can be added or subtracted symbolically when they reflect equal subdivisions of a quantity, they become less reliant on concrete or pictorial models. At this level, students may work with fractions written in improper or mixed number forms; however, they should be aware that applications of fractions to algebra in senior high school more often involve fractions represented in improper form.

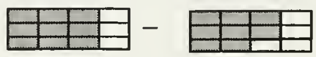
$\frac{3}{5}+\frac{1}{2}$ 

Represent both using the same subdivision of the whole.

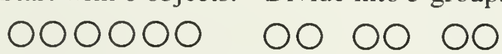
$\frac{6}{10}+\frac{5}{10}=\frac{11}{10}$ or $1\frac{1}{10}$  Combine the two sets to produce a final answer.

$\frac{3}{4}-\frac{2}{3}$ 

Represent both using the same subdivision of the whole.

$\frac{9}{12}-\frac{8}{12}=\frac{1}{12}$  Compare the two sets. The difference represents the final answer.

Adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 8.*

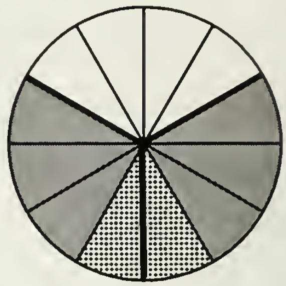
	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	2. When any problem involving fractions is presented, it is important that students first attempt to solve it mentally. If it cannot be solved mentally, students should determine whether an estimate is sufficient or if an exact answer is required. The following are situations where mental computation would be expected: • When denominators are the same, or if the common denominator is easily determined; e.g., $\frac{1}{2}+\frac{1}{4}$, $\frac{7}{10}-\frac{1}{5}$. Such situations occur when one denominator is a multiple of the other. • When a simple fraction is subtracted from or added to a whole number; e.g., $2-\frac{1}{3}$, $4-\frac{2}{3}$, $3+4\frac{2}{3}$. Activities related to mental computation should generally be done for short periods of time. Five to ten minutes at the beginning of a class is usually sufficient. Reproduced with permission from *Atlantic Canada Mathematics Curriculum: Grade 8.* 3. Multiplication of fractions should start with concrete and pictorial models but develop quickly to the symbolic level. Among the simpler combinations to model concretely or pictorially are: • a whole number by a fraction less than one; e.g., $4\times\frac{1}{3}$ uses repeated addition  ($\frac{4}{3}$ or $1\frac{1}{3}$) • a fraction less than one by a whole number; e.g., $\frac{1}{3}\times 6$. Think $\frac{1}{3}$ of 6 Start with 6 objects. Divide into 3 groups.  How many are in each group?

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

- a fraction less than one by any other fraction, especially when the numerator is 1; e.g.:

  $\frac{1}{4}$ of $\frac{2}{3}$
  Divide each $\frac{1}{3}$ into fourths.
  $\frac{1}{4}$ of $\frac{2}{3}$ makes
  $\frac{2}{12}$ or $\frac{1}{6}$.



It should be shown that "of" means multiplication. This may be done by comparing results in such examples as $\frac{1}{4}$ of 8 and $\frac{1}{4} \times 8$.

When work is done at the symbolic level, it should be supported by concrete or pictorial representations. Grid diagrams should also be considered when modelling multiplication. Students should notice that $\frac{1}{2}$ of $\frac{3}{4}$ is represented as $\frac{3}{8}$. By comparing the question with the result, students can start to speculate about a possible algorithm.

Modelling should always be related back to the symbols so that students make the connections clearly; otherwise, the use of models may not help support student understanding of the algorithms. Students should be able to work effectively with multiplication of fractions at the symbolic level in Mathematics Preparation 10.

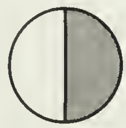
Adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 8.*

4. Students should be able to perform division of fractions at the symbolic level in Mathematics Preparation 10. Nevertheless, it is important that significant time be spent working with concrete and pictorial models. Initial examples for modelling should be chosen carefully and worked through by the teacher prior to instruction. These simpler examples should enable students to derive an algorithm. Situations that are well-suited to modelling with materials include:

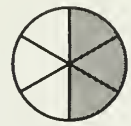
	INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

- a simple fraction divided by a whole number; e.g., for $\frac{1}{2} \div 3$, divide $\frac{1}{2}$ into 3 equal parts. What does each part represent?

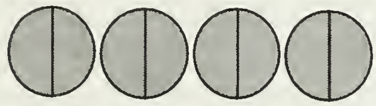


Divide $\frac{1}{2}$ into 3 equal parts.
Answer = $\frac{1}{6}$



Divide into 3 equal parts.

- a whole number divided by a simple fraction; e.g., $4 \div \frac{1}{2}$ asks how many halves there are in 4

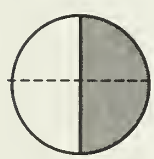


Count the number of halves in 4 objects. Since each object has two halves, $4 \times 2 = 8$. By comparing this to the original question, an algorithm can be developed.

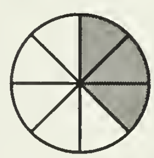
- a simple fraction divided by a simple fraction, where the numerator of the divisor is one and both denominators are the same; e.g., $\frac{5}{6} \div \frac{1}{6}$ asks how many one-sixths there are in $\frac{5}{6}$

How many one-sixths are there in $\frac{5}{6}$?
Answer = 5

- a simple fraction divided by a simple fraction, where the numerator of the divisor is one and the fractions are compatible; e.g., $\frac{1}{2} \div \frac{1}{4}$ or $\frac{3}{8} \div \frac{1}{4}$.



How many quarters are there in $\frac{1}{2}$?
Answer = 2

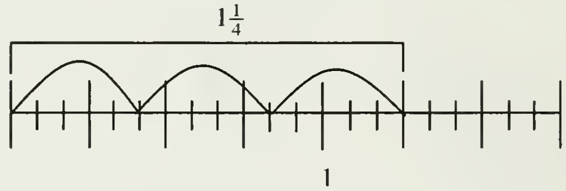


How many quarters are there in $\frac{3}{8}$?
Answer = $1\frac{1}{2}$

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

The number line can also provide a useful model for division. For example, suppose it takes $1\frac{1}{4}$ hours to do 3 chores. How long does it take for each chore, if they all require an equal amount of time? This can be modelled as follows:



Divide each quarter into 3 parts, which makes 15 parts in $1\frac{1}{4}$ hours. There will be 5 parts for each chore, and 12 parts in 1 hour, hence, $\frac{5}{12}$ hour for each chore.

There are two common algorithms for division that can be considered. The common-denominator algorithm involves finding a common denominator and dividing the numerators; e.g., $\frac{4}{3} \div \frac{1}{2} = \frac{8}{6} \div \frac{3}{6} = 8 \div 3 = 2\frac{2}{3}$. This can be modelled concretely using the process described in the fourth bullet above. The more traditional multiply by the reciprocal algorithm involves inverting the divisor and multiplying by it; e.g., $\frac{4}{3} \div \frac{1}{2} = \frac{4}{3} \times \frac{2}{1} = \frac{8}{3} = 2\frac{2}{3}$. It is necessary for students to understand the concept of reciprocal before they work with division using the multiply by the reciprocal algorithm. As a starting point for this algorithm, students can compare the solution of such problems as $8 \div \frac{1}{2}$ and 8×2.

Adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 8.*

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

It is important for students to demonstrate their understanding of simple calculations of fractions **without the use of a calculator**.

Performance

1. Show why the following is an incorrect procedure, through the use of concrete materials or diagrams.

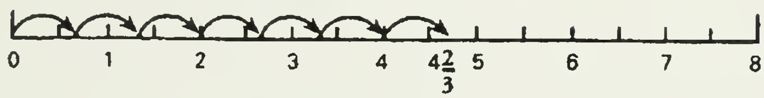
   $\frac{3}{8}-\frac{1}{4}=\frac{3-1}{8-4}=\frac{2}{4}=\frac{1}{2}$

2. Draw a diagram to show why each of the following is true.

   a. $\frac{1}{3}\times 3=1$

   b. $6\times\frac{1}{3}=2$

3. What multiplication sentence is illustrated?



4. Place the numbers 1, 2, 3 and 4 in the boxes to get the smallest possible answer.

   $\frac{\square}{\square}\times\frac{\square}{\square}$

   Try the same type of problem, using the numbers 2, 3, 4 and 5. Choose a different set of four numbers and repeat the activity.

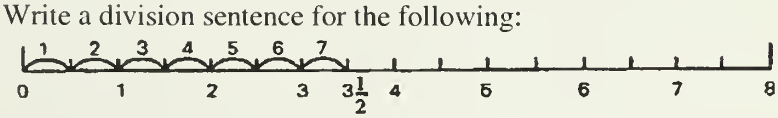
5. a. Use a diagram to explain why each of the following is true.

   $2\div\frac{1}{4}=8$

   $\frac{1}{2}\div 2=\frac{1}{4}$

   b. Compare the solutions in part a with the solutions to 2×4 and $\frac{1}{2}\times\frac{1}{2}$, and discuss any observations.

6. Write a division sentence for the following:



Reproduced with permission from *Atlantic Canada Mathematics Curriculum: Grade 8.*

Paper and Pencil[1]

1. Create three pairs of fractions whose sum is $\frac{1}{2}$.

2. Create three addition and three subtraction sentences with the same result as $\frac{6}{12}+\frac{3}{12}$.

[1] Paper and Pencil questions 5 to 7 are adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 8.*

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	

3. A recipe requires $2\frac{1}{3}$ cups of flour to make 24 muffins. How many cups of flour would be required to make 60 muffins?

4. What might be the value of $\square$, if $3\frac{1}{2} - 1\frac{2}{\square} < 2$?

5. Draw a diagram to show that $\frac{2}{3} \times \frac{1}{4} = \frac{1}{6}$.

6. Michael ordered three extra large pizzas and asked that each pizza be cut into sixteenths. If each person at Michael's party is likely to eat 3 pieces, how many people can the pizzas serve?

7. Lisa has $\frac{3}{4}$ of a large candy bar. She gave $\frac{1}{3}$ of what she had to Shannon.
   a. Explain, in at least two different ways, why Shannon received less than $\frac{1}{3}$ of what would have been a whole bar.
   b. What fraction of a whole candy bar does each girl have?

Interview

1. With the use of concrete materials, explain why the following is incorrect: $\frac{1}{4} + \frac{1}{4} = \frac{2}{8}$.

2. Ask students how they would convince someone that the following is incorrect: $\frac{5}{6} + \frac{5}{8} = \frac{10}{14}$.

3. Ask students:
   a. if an answer can be sixths when they add fourths and thirds, and to justify their response
   b. if an answer can be sevenths when they add fourths and thirds, and to justify their response.

Portfolio

1. Explain how to find the common denominator of two fractions if:
   a. one denominator is a multiple of the other
   b. the two denominators have a factor in common but one is not a multiple of the other.

2. Tell students that Frank works $7\frac{1}{4}$ hours a day for a five-day work week. He works 3 hours on Saturday and is paid time and a half.
   a. What weekly salary would he make at $9.25 per hour?
   b. If Frank has $95.00 per week taken out of his cheque for taxes and union dues, and his father makes him save $\frac{3}{5}$ of his take-home pay for university, how much spending money would be available to Frank on a weekly basis?

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	3. Write two fractional numbers that have a product between the given numbers for each of the following. a. 14 and 15 b. $\frac{1}{3}$ and $\frac{1}{2}$ 4. Complete the following patterns, and extend them for two extra lines. What pattern do you observe? $9 \div 9 =$ $4 \div \frac{1}{2} =$ $9 \div 3 =$ $2 \div \frac{1}{2} =$ $9 \div 1 =$ $1 \div \frac{1}{2} =$ $9 \div \frac{1}{3} =$ $\frac{1}{2} \div \frac{1}{2} =$ $9 \div \frac{1}{9} =$ $\frac{1}{4} \div \frac{1}{2} =$ $\vdots$ $\vdots$ 5. Caitlin decided to make muffins for the school cafeteria. Her recipe requires $2\frac{1}{4}$ cups of flour to make 12 muffins. Caitlin found there were exactly 18 cups of flour in the canister, so she decided to use all of it. a. Ask students how many muffins Caitlin can expect to get. b. The principal of the school liked Caitlin's muffins and asked her to cater the school picnic next year. She will need to produce enough muffins for all 400 students. Ask students how many cups of flour Caitlin will require. Adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 8.*

STRAND: NUMBER (NUMBER OPERATIONS)

GENERAL OUTCOMES

Demonstrate an understanding of and proficiency with calculations on rational numbers.

Decide which arithmetic operations can be used to solve problems and then solve the problem.

SPECIFIC OUTCOME

12. Convert among fractions, decimals and percents to problem solve. [E, PS, T, R] (8–12)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Interactions 7*, pp. 52–54, 104
- *Interactions 8*, pp. 134–154
- *Mathpower 7*, pp. 52–55, 150–161
- *Mathpower 8*, pp. 24–27, 116–117, 119–121, 126–130, 132–133, 138–139, 143
- *Minds on Math 7*, pp. 144–177
- *Minds on Math 8*, pp. 172–197
- *Minds on Math 9*, pp. 40–43
- *TLE 7*, Fraction Conversion, Student Refresher pp. 10–11, Teacher's Manual pp. 32–35
- *TLE 7*, Decimal Conversion, Student Refresher pp. 12–13, Teacher's Manual pp. 36–39
- *TLE 8*, Problem Solving with Fractions, Student Refresher pp. 24–25, Teacher's Manual pp. 60–63

Previously Authorized Resources
- *Journeys in Math 8*, pp. 152–155, 198–203
- *Journeys in Math 9*, pp. 284–285
- *Math Matters: Book 2*, p. 28

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	• Final answers expressed as fractions should always be reduced to lowest terms. • Estimation of the answer should always be stressed. Students should be able to use the equivalent relationships of $\frac{1}{2} = 50\%$, $\frac{1}{4} = 25\%$, $\frac{1}{5} = 20\%$ and $\frac{1}{10} = 10\%$ to approximate other fractions and percentages. • Emphasize the importance of using conversions in a problem-solving context. Encourage students to relate their conversions to areas of personal interest, such as test marks, batting averages, local sales, sales taxes or deductions from pay cheques.

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

Paper and Pencil

1. On Alice's last visit to the fitness centre, she spent 20% of her time in the pool, 15% on the stationary cycle, 25% in the racquetball court, 25% on the jogging track, and the remainder of her time in the locker room.

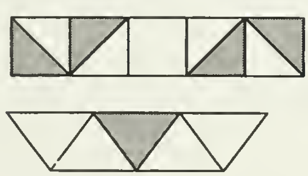
   a. What fraction of the time did she spend in the locker room?
   b. If Alice arrived at the fitness centre at 1:00 p.m. and left at 4:30 p.m., how many minutes did she spend in the pool?

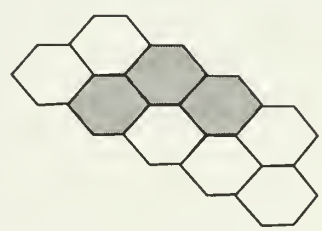
2. A ski shop offers an end-of-season clearance sale with the following reductions:

Day of the Sale	Percentage Reduction Off Original Price
1	10
2	20
3	25
4	30
5	45
6	50

   a. What fraction of the regular price is the sale price on the third day of the sale?
   b. If the percentage reduction was off the previous day's sale price, instead of the original price, approximately what fraction of the original price is the sale price on the third day?

3. Express the shaded region as a:
   a. fraction
   b. decimal
   c. per cent of the whole.





4. The hockey team at Centre Point High School won 12 of the 20 games it played.

   a. What percentage of the games did it win?
   b. If three of the games were tied, what percentage of the games did it lose?

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	5. Tom's spending for the weekend is given in the following table. Expense | Amount: Pizza — $15 Video — $4 Jeans — $60 Gasoline — $30 Swimming — $18 Hockey game — $25 Movie — $8 Express the amount of each expense as: a. a fraction of the total (reduced to lowest terms) b. a percentage of the total. **Journal Entry** 1. Explain the steps for converting: • a fraction or decimal to a per cent • a per cent to a fraction or decimal • a decimal to a fraction. **Portfolio** 1. Chart your daily schedule according to the amount of time, in hours, spent at each activity. Include at least seven activities, but limit the number of activities to no more than 10. Determine the time spent on each activity as a fraction of 24 hours. Then calculate the percentage of the day spent doing your two most time-consuming activities.

STRAND: NUMBER (NUMBER OPERATIONS)

GENERAL OUTCOMES	Demonstrate an understanding of and proficiency with calculations on rational numbers. Decide which arithmetic operations can be used to solve problems and then solve the problem.
SPECIFIC OUTCOME	13. Estimate and calculate operations, on rational numbers. [E, PS, T] (8–10)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Interactions 8*, pp. 198–215
- *Interactions 9*, pp. 87–90
- *Mathpower 8*, pp. 63–71, 74–75
- *Minds on Math 8*, pp. 283–295
- *Minds on Math 9*, pp. 32–36
- *TLE 8*, Adding and Subtracting Rational Numbers, Student Refresher pp. 26–27, Teacher's Manual pp. 64–67
- *TLE 9*, Rational Numbers and Problem Solving, Student Refresher pp. 20–21, Teacher's Manual pp. 52–55

Previously Authorized Resources

- *Journeys in Math 9*, pp. 68–75
- *Math Matters: Book 2*, p. 82

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes Use consistent language when working with integers. Positive and negative refer to signed numbers, whereas plus and minus refer to operations.	Review Order of Operations • Operations within brackets are performed first. • Operations with exponents are performed next. • Multiplication and division are performed next, in the order they occur from left to right. • Addition and subtraction are performed last, in the order they occur from left to right. B rackets E xponents DM divide and/or multiply AS add and/or subtract This is a memory device for remembering order of operations. Discuss why there is a need for a specific order. Note that this is not always obvious but involves the need for consistency. Calculator keying must be checked.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	1. Solve and discuss the answers to the following: a. $6 + (^-2) \times (^-4) \div (^-1)$ b. $[6 + (^-2)] \times (^-4) \div (^-1)$ c. $6 + (^-2) \times [(^-4) \div (^-1)]$ 2. Solve: a. $6 + (^-3) \times (^-22)$ b. $^-7 + 2 \times {}^-14$ c. $[^-12 + {}^-3] \times {}^-16 + {}^-4$ 3. Write an expression for each of the following, then solve. a. Multiply the sum of $^-35$ and 42 by $^-4$. b. Subtract $^-16$ from $^+18$, then divide by 4. c. Add the product of $^-7$ and $^-24$ to $^+52$. d. Divide $^+81$ by the product of $^-3$ and $^+3$. e. Subtract the sum of $^-42$ and $^-9$ from the product of $^-17$ and $^+5$. 4. Meadowview High put on a school dance. The admission charge was \$6.50 per person. The cost of the DJ was \$750.00, and the cost of decorating the gymnasium was \$200.00. If 180 people attended the dance, what was the profit?

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Portfolio** 1. Have students use their calculators to answer the following question. Mary found the attendance reports for the first nine hockey games of the season to be 2787, 2683, 3319, 4009, 2993, 3419, 4108, 3539 and 4602. Tickets were sold for $12.75 each. The expenses for each game were as follows: Stadium costs: $15 000 Employee costs: 100 employees at $12.00 per hour for 6 hours per game Miscellaneous costs: $7000 per game What was the total profit for the nine games? Adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 7.* 2. Collect and solve contest skill-testing questions.

STRAND: NUMBER (NUMBER OPERATIONS)

GENERAL OUTCOMES

Demonstrate an understanding of and proficiency with calculations on rational numbers. Decide which arithmetic operations can be used to solve problems and then solve the problem.

SPECIFIC OUTCOMES

14. Solve problems involving multiple steps and multiple operations, and accept that other methods may be equally valid. [PS] (5–13)
15. Use a variety of methods to solve problems, such as drawing a diagram, making a table, guessing and testing, using objects to model, making it simpler, looking for a pattern, using logical reasoning and working backward. [PS, R, T, V] (6–14)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Interactions 7*, pp. 214–215, 208–209
- *Interactions 8*, p. 108
- *Mathpower 7*, pp. 12–13
- *Mathpower 9*, pp. 88–89
- *Minds on Math 7*, pp. 316–320
- *TLE 7*, Order of Operations, Student Refresher pp. 26–29, Teacher's Manual pp. 66–69
- *TLE 7*, Patterns and Relations, Student Refresher pp. 44–45, Teacher's Manual pp. 100–103
- *TLE 9*, Rational Number and Problem Solving, Student Refresher pp. 20–21, Teacher's Manual pp. 52–55

Previously Authorized Resources

- *Math Matters: Book 2*, p. 37 and integrated throughout the grades

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Students should be given the opportunity to work with rational numbers (positive and negative) in problem-solving situations involving real-life experiences relevant to their age group. Relate addition and subtraction of integers to real-life situations, such as football. If a team gains 5 yards and then loses 8 yards on the next play, what is the total gain/loss? Some background information on context, such as stock market or altitudes, may be required.

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

Example: Josée has $465.97 in her bank account. She writes cheques for $72.39, $16.95 and $173.00. Her GST rebate of $69.20 is automatically deposited in her account. What is her current balance?

$465.97 – $72.39 – $16.95 – $173.00 + $69.20 = $272.83

Her current balance is $272.83.

It is also important to stress the concept that there are many different methods of solving a problem and that all are acceptable as long as they are mathematically correct.

Obviously a farmer wouldn't really use this method of determining how many animals he has, but most students will enjoy solving the problem.

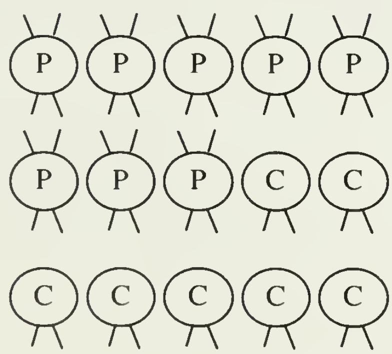
1. A farmer raises pigs and chickens. He has 15 animals in all, and the total number of legs of these animals is 46. How many of each does he have?

   Strategies:
   A. Use a Chart

# of Pigs	# of Chickens	Total # of Legs
5	15 – 5 = 10	5 × 4 + 10 × 2 = 40
7	15 – 7 = 8	7 × 4 + 8 × 2 = 44
8	15 – 8 = 7	8 × 4 + 7 × 2 = 46

   There are 8 pigs and 7 chickens.

   B. Diagram—Each circle represents an animal; each line segment represents a leg.
   Each animal has at least 2 legs; therefore, 30 legs in total if all chickens. Add 2 legs to each animal until the total is 46 legs.



	INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

C. Algebra
Let p equal the number of pigs. Let $15 - p$ equal the number of chickens. Therefore:

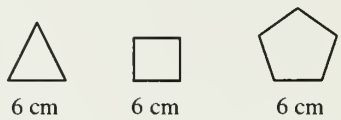
$$4p + 2(15 - p) = \text{total number of legs}$$
$$4p + 30 - 2p = 46$$
$$2p = 16$$
$$p = 8$$

There are 8 pigs and 7 chickens.

Understanding the problem should also include an understanding of the solution. Students should have some concept of a reasonable answer, and refer to this throughout the solving of the problem.

Supply a variety of manipulatives; e.g., grid paper, algebra tiles, cube-a-links and pattern blocks, and encourage students to model the problem using concrete materials if applicable.

2. The following figures were made with toothpicks that are 6 cm long.



a. Look at the pattern formed by the toothpick figures. How are the figures the same?
b. Sketch/create the next three figures in the pattern.
c. Calculate the perimeter of each figure and use this information to complete the table below:

number of sides	1	2	3	4	5	6	7	8
perimeter								

d. Graph the data from part c, using number of sides for the horizontal axis and perimeter for the vertical axis. Describe the pattern in the table and the graph.
e. Suppose this pattern continues. What is the perimeter of a polygon with 75 sides?

Strand: Number (Number Operations)
Specific Outcomes: 14. Solve problems involving multiple steps and multiple operations, and accept that other methods may be equally valid. [PS] (5–13)
15. Use a variety of methods to solve problems, such as drawing a diagram, making a table, guessing and testing, using objects to model, making it simpler, looking for a pattern, using logical reasoning and working backward. [PS, R, T, V] (6–14)

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

Performance

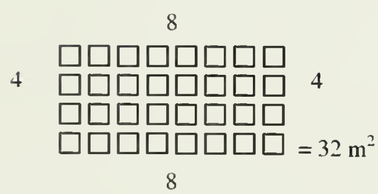
1. Lisa wants to build a rectangular dog run. What is the maximum area that can be enclosed if she has 24 linear metres of chain link fence? (Restrict your answer to the use of whole numbers.)

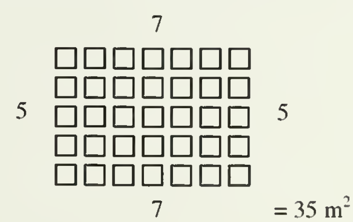
   a. Use a chart.

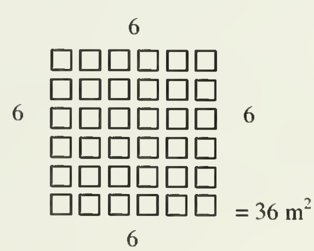
Width (m)	Length (m)	Area (m^2)
1	11	11
2	10	20
3	9	27
4	8	32
5	7	35
6	6	36
7	5	35

   The maximum area is 36 square metres.

   b. Use grid paper or unit algebra tiles. Each 1 × 1 square represents 1 square metre.
   - Cut the paper into 1 × 1 squares, then rearrange into rectangles—20 squares will be needed for the perimeter of the rectangles, since corner squares are counted for both length and width.
   - The rectangles are filled in with 1 × 1 squares, and the total number of squares in the rectangle will represent the area. The maximum area can then be determined.







Specific Outcomes: 14. Solve problems involving multiple steps and multiple operations, and accept that other methods may be equally valid. [PS] (5–13)

15. Use a variety of methods to solve problems, such as drawing a diagram, making a table, guessing and testing, using objects to model, making it simpler, looking for a pattern, using logical reasoning and working backward. [PS, R, T, V] (6–14)

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

Encourage students to use different lengths for the side of the square.

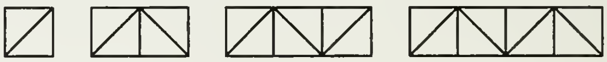
2. Students require grid paper and three different-coloured pencils. They are asked to outline a square with side lengths of at least four squares. Then have students colour the squares according to the following criteria.
   Yellow – all sides adjacent to another square
   Green – 3 sides adjacent to another square
   Blue – 2 sides adjacent to another square
   Example:

B	G	B
G	Y	G
B	G	B

   a. What colour are the interior squares?
   b. Use words to describe the location of blue and green squares.
   c. Write a rule for the number of yellow, green or blue squares, given the length of the square as "n."
   d. Is there a square that contains 250 green squares? 252 green squares? Explain.
   e. Is there a square with 400 yellow squares? 1000 yellow squares? Explain.
   f. Is it possible to have a square with eight blue squares?

Paper and Pencil

1. Each of the following rectangles is divided into squares, which in turn are divided into two triangles.

 …

   a. Complete the table according to the pattern illustrated above.

Length of Rectangle	1	2	3	4	5	6
Number of Triangles	2	5		11		

   b. How many triangles would there be in a rectangle of length 20? Length 100?

2. Cathy earned and saved $1000 during the summer holidays. She does not plan on working during the school year. She budgets to spend $40 per week.
   a. Copy and complete the table below, assuming that Cathy sticks to her budget.
   b. Draw a graph to show how much Cathy has left at the end of each week.

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

c. Write an equation that describes the amount of money she has left after t weeks of spending.
d. When will Cathy run out of money?

Week	Remaining Money
1	$960
2	$920
3	
6	
10	
	$480
	$320
	$200

3. Mount Everest is 8 850 m above sea level. Bernardin Cave in France is 10 344 m lower. What is the altitude of Bernardin Cave?

4. Land makes up three-tenths of the world's surface. One third of the land was once forest. What fraction of the world was once forest?

5. The value of Videoquest stock on Monday of a certain week is given in the table, along with the daily change for the week. Copy and complete the table.

	Opening Value	Change	Closing Value
Monday	10.00	–1.05	
Tuesday		+0.80	
Wednesday		+0.30	
Thursday		–0.45	
Friday		–0.25	

a. What is the closing value of the stock on Friday?
b. What would it cost to purchase 100 shares at opening time on Tuesday?
c. How much profit (or loss) would be realized if 200 shares were purchased at opening time on Monday and then sold at closing time on Thursday?
d. When during the week would it have been best to purchase stock?

STRAND: NUMBER (NUMBER OPERATIONS)

GENERAL OUTCOME Illustrate and apply the concepts of rates, ratios, percentages and proportion to solve problems.

SPECIFIC OUTCOME 16. Understand the meaning of rate, ratio, percentages and proportion and apply these concepts to solve problems. [E, PS, T] (8–12)

MANIPULATIVES

- Base-ten blocks

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Interactions 7*, pp. 152–174
- *Interactions 8*, pp. 125–163
- *Mathpower 7*, pp. 150–161, 174–189
- *Mathpower 8*, pp. 79–109
- *Minds on Math 7*, pp. 146–173, 412–441
- *Minds on Math 8*, pp. 118–167, 174–193
- *Minds on Math 9*, pp. 40–43
- *TLE 7*, Rate and Ratio, Student Refresher pp. 38–39, Teacher's Manual pp. 88–91
- *TLE 7*, Percent, Student Refresher pp. 36–37, Teacher's Manual pp. 84–87

Previously Authorized Resources
- *Journeys in Math 8*, pp. 168–186, 198–222
- *Journeys in Math 9*, pp. 268–294
- *Math Matters: Book 2*, pp. 200–219

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes The focus should be on an intuitive understanding of per cent.	**Per Cent**❶ Number sense for per cent should be developed through the use of benchmarks: • 99% is almost all • 49% is almost half • 10% is not very much • 1% is very small in relation to total.

❶ Information on Per Cent is adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 7* and *Atlantic Canada Mathematics Curriculum: Grade 8*.

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

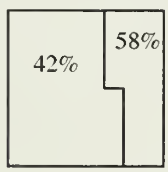
Visual representations should not be restricted to circular form.

Discussion should also focus on the contexts in which 1% would be considered high and the contexts in which 99% would be considered low. For example, consider the mercury content of fish that might be hazardous to humans versus the success rate of an air traffic controller.

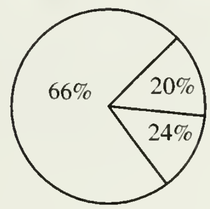
- Students should relate to per cent visually. Ask students to estimate the per cent that the shaded portions of the following diagrams represent.

Ask students what is incorrect about each of the following:

a.



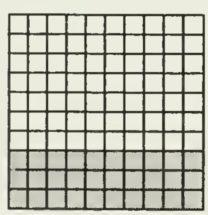
b.



c. Sarah wrote a test and had 8 questions right and 10 questions wrong. Sarah announced, 8 out of 10 is 80%—that's not a bad grade!

Students should be able to determine both accurate and approximate per cents. That is, students should be able to give an accurate value for the percentage shaded in diagram A below; and they should be able to estimate the percentage shaded in diagram B.

A.



B.

Students should make immediate connections between certain percentages and their fraction equivalents; e.g., 25%, 50%, 75%, and 20%, 30%, 40%. They should also be encouraged to recognize that per cents such as 51% or 49% are close to $\frac{1}{2}$ and, therefore, use $\frac{1}{2}$ for estimation purposes.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

Complete the table.

Percentage	Fraction
	1/5
	1/3
	1/4
	1/2
	2/3
	2/5

When exact answers are required, students should be able to employ a variety of strategies in calculating per cent of a number, including:

- changing per cent to a decimal and multiplying; e.g.:
  12% of 80 = 0.12 × 80 = ?
- changing to a fraction and dividing; e.g.:
  25% of 60
  $= \frac{1}{4} \times 60$
  $= 60 \div 4$
- using the per cent key on a calculator.

Another technique that can be used to help students convert fractions to per cent, or per cent to fractions is to remember

$\boxed{\frac{\text{part}}{\text{whole}}}$ ← numerator
← denominator

or $\boxed{\frac{\text{is}}{\text{of}}}$.

Example

1. 12% of what number is 17?

   part → $\frac{12}{100} = \frac{17}{x}$ ← is
   whole → ← of

2. What per cent of 25 is 8?

   $\frac{x}{100} = \frac{8}{25}$ ← is
   ← of

Evaluate:
18% of 40
25% of 320
40% of 16
90% of 200

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	

Students should be able to compute 7%, 10%, 25%, $33\frac{1}{3}$%, 50% and 1% mentally. This skill can be used to help with other mental computations.

Practice should also include per cents less than one and greater than 100.

Students will sometimes encounter occasions involving two percentages. Students should realize that, when combining percentages, they cannot add the percentages directly; e.g., if a tennis racket was already on sale for 20% off and the store announced a sale that read, "30% off all items in the store, including items already on sale," students should explore what happens when the two percentages are combined—compare calculating a 50% discount, versus taking off 20% followed by 30%.

Students should also be made aware that problems involving a discount can be solved in more than one way. For example, the discounted price can be determined either by finding 20% of the original price and subtracting or, more efficiently, by finding 80% of the original price.

In general, percentage increase or decrease is found using the following:

$$\text{Percentage increase} = \frac{\text{increase}}{\text{original amount}} \times 100\%$$

$$\text{Percentage decrease} = \frac{\text{decrease}}{\text{original amount}} \times 100\%$$

Students have worked with per cent in previous grades. Per cents greater than 100, however, can be somewhat abstract for many students.

An increase of 25% means that the final is 125% of the original; e.g., after a 200% increase, a \$50 item has a price of:
\$50 + 200% of 50
= \$50 + 100
= \$150

Questions

1. Two-eighths of the tickets were sold for a concert held in a concert hall that has a 925-seat capacity. What percentage of the total capacity was used?

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	2. The manager of a concert hall indicated that, in order to make a profit, the hall must be filled to at least 70% capacity or else the price of each ticket increases. The seating capacity is 1200, and advance ticket sales are at 912. Will a profit be made based on the number of tickets sold in advance sales?

3. A $400 set of tires is marked up 15%, and 7% GST is added to the marked up price. What is the selling price, including tax?

4. While there are many forms that can be used to express most numbers, certain forms are associated with various contexts or situations. Ask students which form is typically associated with each of the following:
   a. a special end-of-season sale at a clothing store (percentage)
   b. the batting average of a baseball player (decimal)
   c. the part of a cup that is used in a typical recipe (fraction)
   d. the teeth on the wheels and the gears in a bicycle (ratio)
   e. the sales tax (percentage)
   f. test scores (percentage)

5. Ask students to estimate a per cent that is a close approximation for each of the following and to indicate why their estimate is larger or smaller than the exact value. (They do not need to find the exact value to do this.)
   a. $\frac{7}{11}$
   b. 4:9
   c. $\frac{6}{13}$
   d. 7:16

6. Complete the following table. Assume Provincial Sales Tax (PST) is 6% and GST is 7%.

Item	Price	PST	GST	Total Cost
motorbike	$1480.00			
shoes	$89.99			
CD player	$124.50			

7. Have students work in pairs. Each student first works individually to create three problems, using a newspaper flyer. This student then solves these problems on a separate sheet of paper. Partners swap problems and solve them. Solutions are checked by the person who originally created the problems. When differences in solutions occur, both students work together to try to determine the source of error. (Sometimes the source of error may be a vaguely worded question. This can provide some information for further discussion with the small group or the whole class.)

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Mental Mathematics** 1. If 2% of a certain number is 8: a. what would 10% of the number be? b. what is the number? 2. Sarah has a savings account that earns $\frac{1}{2}$% interest monthly. Jane has a savings account where she earns $5\frac{3}{4}$% annually. Who do you think would have more money in the bank at the end of one year if they both start with the same amount? Why? **Paper and Pencil**[1] 1. Mr. Jones bought a mining stock at \$35. Two weeks later he sold it for \$105. What was the percentage increase? 2. The Canadian dollar was valued at 70.0¢ US on Friday. On Monday, the opening value was 68.5¢ US. What was the percentage decrease? 3. Mack's Sound Emporium purchased CD players for \$129 per unit and is planning to sell them for \$195.99. It purchased 150 units. a. What is the percentage increase (markup) per unit? b. How much can Mack expect to make if he sells all the units? c. After four weeks, Mack realizes that the CD players are not selling as fast as he hoped, so he puts them on sale for 20% off. If he sells 56 units for the duration of the sale, how much money will he make on the items sold? 4. John's father said, "In my youth I could buy a chocolate bar and a soft drink for 20¢." What would be a typical cost for these items today? Estimate the percentage increase this represents. 5. A politician was elected with 2145 votes at a convention. If she received 58% of the votes cast, about how many votes were cast? (The solution mentally might be as follows: 60% of □ = 2100; guess 3000; 60% of 3000 = 1800; guess 4000; 60% of 4000 = 2400. Since 2100 is exactly halfway between the two guesses, a third guess might be 3500. Since an exact answer is not required, this seems like a reasonable estimate.) 6. Suits selling regularly for \$185.00 were marked down by 25 per cent. To further improve sales, the discount price was reduced by another 15 per cent. What was the final selling price? What was the total per cent discount on the original price?

[1] Paper and Pencil questions 1 to 3 are adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 8.*

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

This question assumes students have a knowledge of circle graphs and that 360° = 1 circle.

7. In reading a circle graph, Sarah realized that the sections of the graph did not contain any numbers or per cents. She decided to use the angle measures to help read it. The section that represented the number of red cars looked as if it was an angle of about 90°.
   a. Explain how to find what percentage of the cars are red.
   b. It was more difficult to estimate the number of degrees the blue cars represented, so Sarah used a protractor and found the angle to be exactly 145°. What percentage of the cars are blue?
   c. Suppose the circle graph represents the colours of the cars that pass an intersection during a one-hour period. Based on the information provided, if 400 cars passed this intersection, how many would you expect to be blue and how many would you expect to be red? How many would you expect to be neither blue nor red?

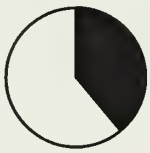
8. A store has a NO GST sale. Darcy purchased a skirt priced at $39.99. When she paid for it, the clerk first subtracted 7% to get a new price and then added 7% GST to this new price. Is this a fair way to calculate the price? Why would a store use this practice?

9. McDunphy's Burger Heaven has a sale on hamburgers. A hamburger is half price when you buy a medium drink and a medium fries. The regular prices are as follows: hamburger $2.30, medium drink $1.29 and medium fries $1.39. What is the actual percentage off the regular price when you take into account what must be purchased to take advantage of the sale?

Performance

1. Tell students that a flat from a set of base-ten blocks represents 100% of something. Ask them to use base-ten blocks to represent
   a. 110%
   b. 125%
   c. 200%
   d. 450%

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Portfolio** 1. Ask students to draw a rectangle and a triangle of any dimensions and to solve the following. a. Find the area and the perimeter of each figure. b. Increase the dimensions of each figure by 30%, and find the new perimeter and area. c. Decrease the dimensions of each of the original figures by 40%, and find the new perimeter and area. d. Find the ratio of new perimeter to original perimeter and the ratio of new area to original area for each of parts b and c. What do you notice? Adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 8.* 2. Tell students that Sarah found out that the new car she bought would depreciate in value by 20% per year. Sarah paid $20 000 for the car and planned to keep it for three years. She wanted to find the car's value at the end of three years and asked a friend to help. They decided to do their calculations independently and then compare answers. Sarah's answer was $10 240, but her friend's answer was $8000. a. Ask students how each of the answers was obtained. b. Ask them who they think is correct and to explain their choice. **Interview** 1. Ask students to: a. explain why 70% is not a good estimate for 35 out of 80 b. explain how to estimate the percentage when a test score is 26 correct out of 55 c. change each of the following to a per cent, mentally, and explain their thinking: $\frac{2}{5}, \frac{4}{25}, \frac{6}{50}, \frac{7}{20}, \frac{1}{3}$ d. estimate the per cent for each of the following, and explain their thinking: $\frac{7}{48}, \frac{5}{19}, \frac{7}{26}$ e. indicate what per cent of a book is left to read if they have read 60 out of 150 pages, and ask them to explain their thinking.

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Journal Entry** 1. A certain chemical is dangerous to humans if it is found in the water supply at more than 275 parts per million. a. Ask students what per cent this represents. b. Ask them what per cent of the chemical would represent a danger level in 1000 L of water. 2. Ask students to critique the following situation and to explain why the reasoning is flawed. Jim found out that on his test the ratio of questions answered correctly to questions answered incorrectly was 12:13. He concluded that he should get a very good grade. 3. Estimate the percentage that is shaded for each of the following diagrams. Explain your reasoning.  

STRAND: NUMBER (NUMBER OPERATIONS)

GENERAL OUTCOME	Illustrate and apply the concepts of rates, ratios, percentages and proportion to solve problems.
SPECIFIC OUTCOMES	16. Understand the meaning of rate, ratio, percentages and proportion and apply these concepts to solve problems. [E, PS, T] (8–12) 17. Express rates and ratios in equivalent forms. [CN, PS, R] (8–15)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Interactions 7*, pp. 152–174
- *Interactions 8*, pp. 125–163
- *Mathpower 7*, pp. 150–161, 174–189
- *Mathpower 8*, pp. 79–109
- *Minds on Math 7*, pp. 146–173, 412–441
- *Minds on Math 8*, pp. 118–167, 174–193
- *Minds on Math 9*, pp. 40–43
- *TLE 7*, Rate and Ratio, Student Refresher pp. 38–39, Teacher's Manual pp. 88–91
- *TLE 7*, Percent, Student Refresher pp. 36–37, Teacher's Manual pp. 84–87

Previously Authorized Resources
- *Journeys in Math 8*, pp. 168–186, 198–222
- *Journeys in Math 9*, pp. 268–294
- *Math Matters: Book 2*, pp. 200–219

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes Students should understand that proportion is a statement of equality between two ratios. The emphasis should be on developing proportional reasoning. This topic is rich in real-world connections. It is when proportions are embedded in such real-life contexts as scale models, altering recipes and comparison shopping that students relate to the process used in finding unknown values. The study of scale is an important application of work with ratio and proportion, and connects well with geometry and enlargements and reductions. Adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 8.*	**Proportion** A *proportion* is a statement of equality of two ratios. Example: • The florist has a special on bouquets, 1 rose to 2 carnations. You want to buy one of these bouquets for your mother. If you want 6 roses in the bouquet, how many carnations will it include? To solve the question of the flowers, students would set up a proportion statement such as the following: Rose : Carnations Roses : Carnations 1 : 2 6 : $\boxed{?}$

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

Students should be very secure in using this type of notation before they are introduced to the fraction symbol for proportion.

Although proportions and equivalent fractions appear to be the same thing, they are not. Equivalent fractions are different symbols for the same amount. If you colour $\frac{1}{2}$ a piece of paper and fold it so that it now shows $\frac{2}{4}$, you still have the same amount coloured $\left(\frac{1}{2}=\frac{2}{4}\right)$. On the other hand, if you buy 2 bouquets of flowers and one has 1 daisy and 2 roses and the other has 2 daisies and 4 roses, the total number of flowers is different but the ratio of daisies to roses is the same (1 : 2 = 2 : 4) and therefore proportional.

Reproduced, by permission, from Manitoba Education and Training. *Grades 5 to 8 Mathematics: A Foundation for Implementation*. Winnipeg, MB: Manitoba Education and Training, 1997.

Sample Questions

1. If stereo speakers are to have good acoustics, the ratio of their depth to their width to their height is 1 to 2 to 3. If a speaker is 90 cm high, how deep and wide should it be to have good acoustics?

   Because 3 was multiplied by 30 to get 90, multiply the other terms by 30.

   $$\begin{array}{ccccc} \text{depth} & : & \text{width} & : & \text{height} \\ 1 & : & 2 & : & 3 \\ \downarrow & & \downarrow & & \downarrow \times 30 \\ 30 & : & 60 & : & 90 \end{array}$$

   The speaker should be 30 cm deep and 60 cm wide.

2. The ratio of the length of an adult's small intestine to the length of the adult's large intestine is about 4 to 1. If a person has a total of 7.5 m in small and large intestines, how long is the small intestine? How long is the large intestine?

   Method 1

   ratio of small : large : total is 4 : 1 : 5

   $\bigcirc : \square : 7.5$

   Since $5 \times 1.5 = 7.5$, multiply all values by 1.5 to get 6 : 1.5 : 7.5

   Method 2

   x = the length of the large intestine

   $4x$ = the length of the small intestine

   $$\begin{aligned} 4x + x &= 7.5 \\ 5x &= 7.5 \\ x &= 1.5 \end{aligned}$$

   The large intestine is 1.5 m and the small is (4×1.5) or 6 m.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	3. Theoretically, the ratio of the height a person can jump on the moon to the height a person can jump on Earth is 6 to 1. If a pole-vaulter can jump 4.5 m on Earth, how high could she jump on the moon? Moon : Earth 6 : 1 ☐ : 4.5 Multiply both values by 4.5 to get 6 : 1 = 27 : 4.5. She can jump 27 m on the moon. 4. A bag contains red marbles, white marbles and blue marbles. The ratio of the red marbles to white marbles to blue marbles is 2 to 3 to 4. If there are 12 blue marbles in the bag, how many red marbles are there? How many white marbles are there? 5. The amount of gold in jewelry is measured in karats (K). This measure is a ratio expressed as a single number. The second term is understood to be 24. For example, the mark of 10 K means that the ratio of the mass of gold in the jewelry to the total mass of the metals is 10 to 24. If a ring is marked 14K and the ring (without any stones) has a mass of 72 g, what is the mass of the gold in the ring? 6. Sterling silver is an alloy of silver and copper in the ratio of 37 to 3. If a sterling silver goblet has a mass of 800 g, what is the mass of silver in the goblet? What is the mass of copper in the goblet? 7. If it takes 2.2 hours to write a seven-page essay, how long might it take to write an 18-page essay? 8. If the scale of a map is 1 : 50 000, and two towns are located 9 cm apart on a map, what is the actual distance between the two towns? **Rate** Rate is a quotient used to compare two measures of **different** units; e.g., kilometres per hour. This topic is rich in real-life, problem-solving opportunities, such as rate of pay, cost per unit and rate of travel.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Sample Questions 1. If a 3-pack of juice boxes costs \$1.09, what would 12 juice boxes cost? (Students may use $3 \times 4 = 12$, so the cost is $\$1.09 \times 4 = \$4.36$.) 2. Find the cost of 18 chocolate bars, if 10 chocolate bars cost \$2.19. $\frac{\$2.19}{10} = \frac{x}{1}$ $x = 21.9¢ \cong 22¢$ So the cost of 18 chocolate bars is $22 \times 18 = \$3.96$. 3. Ask students to find how long it would take to produce an 18-page essay if they can produce a five-page essay in 2.2 hours. Students might do the following: $\frac{2.2}{5} = \frac{x}{18}$ $\frac{2.2 \times 18}{5} = x$

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	

Paper and Pencil[1]

1. Study each of the proportions, and estimate which of *a*, *b*, *c* or *d* represents the largest value. Solve to verify your estimate.

   a. $\frac{3}{7} = \frac{a}{28}$　　b. $\frac{b}{9} = \frac{3}{4}$

   c. $\frac{5}{c} = \frac{15}{33}$　　d. $\frac{5}{8} = \frac{23}{d}$

2. Pat has three cats for every five dogs in her kennel.
   a. In September, Pat had 25 dogs. How many cats did she have?
   b. In January, Pat had 48 cats and dogs altogether. How many of Pat's animals were dogs?

3. What is the scale of a map if 7.2 cm in the map represents a distance of 1800 km?

4. In planning his across-Canada tour, Orville estimated he could bike from Victoria to Halifax, a distance of approximately 6050 km, in 57 days. Calculate his rate of travel to the nearest kilometre per day.

5. If it takes Hugh 3.5 hours to drive from home to Calgary, a distance of 280 km, what is his average speed in kilometres per hour?

6. Joe earns $420 in a 40-hour work week. Calculate his hourly wage.

Interview[1]

1. Tell students that when making lemonade Sue uses 5 scoops of powder for 6 cups of water, and Sarah uses 4 scoops of powder for 5 cups of water. Ask students the following:
   a. Are the situations proportional to each other? Explain why or why not.
   b. In which situation is it likely the lemonade will be more flavourful? What assumptions did you make?

[1] Paper and Pencil questions 1 and 2, and Interview questions 1, 2 and 4 are adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 8*.

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	2. Ask students to discuss in their journals whether or not the following could be solved using a proportion: David is 6 years old and Ellen is 2 years old. How old will Ellen be when David is 12 years old? 3. Ask students to explain whether or not, in the proportion $3 : 8 = 17 : x$, x can be a whole number. 4. Ask students to explain why 1 : 20 000 000 is another way to describe the ratio of 1 cm representing 200 km on a map. **Portfolio** 1. A statue of Lord Strathcona was made from a model. The height of the model was 25 cm. Ask students to find the height, in metres, of the statue if it was made using a scale of 1:15. 2. What scale would have been used if a 90 m building is 15 cm tall in a diagram?

STRAND: NUMBER (NUMBER OPERATIONS)

GENERAL OUTCOME Apply exponent laws to solve problems.

SPECIFIC OUTCOME 18. Use exponent laws to evaluate expressions with numerical bases. [PS, R, T] (9–9)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Interactions 9*, pp. 28–36
- *Mathpower 8*, pp. 4–7
- *Mathpower 9*, pp. 20–21, 26–29
- *Minds on Math 8*, pp. 458–461
- *Minds on Math 9*, pp. 276–290
- *TLE 9*, Evaluating Powers and Expressions, Student Refresher pp. 16–19, Teacher's Manual pp. 44–51

Previously Authorized Resources

- *Journeys in Math 9*, pp. 96–101

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Students should first understand the meaning of powers; e.g., 2^3, 4^4. Instruction should be designed so students discover rules/relationships and verify their discoveries. Otherwise students memorize "rules" without understanding why they work. Development of the laws through easily worked examples, should be the emphasis. Examples 1. $2^3 \times 2^4 = (2 \times 2 \times 2) \times (2 \times 2 \times 2 \times 2) = 2^7$ This would prompt students to express the law for multiplication of powers with the same base. 2. Have students calculate $(5^2)^2$, $(2^2)^4$ and $(3^2)^3$, and compare the results with 5^4, 2^8 and 3^6 to establish the power of a power rule. 3. Have students calculate $3^4 \div 3^2$ and $4^5 \div 4^3$ as follows. $\frac{3\times 3\times 3\times 3}{3\times 3} = \frac{81}{9} = 9 = 3^2$; $\frac{4\times4\times4\times4\times4}{4\times4\times4} = \frac{4\times4}{1} = 16 = 4^2$ Students can generalize the results to form a rule for dividing powers with the same base. They can verify with other examples.

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Paper and Pencil** 1. Dale has a stereo system with an amplifier as well as a preamplifier. The preamplifier boosts the signal by a factor of 10^5, and the amplifier boosts the signal by a factor of 10^4. Calculate the factor by which the signal is boosted after it has passed through both amplifiers, if this factor is a product of the two individual factors. 2. It has been estimated that each galaxy contains 10^{11} stars. If there are about 10^{11} galaxies in the universe, approximately how many stars are there in total? Explain why. (You can express the number of stars as $10^{11} \times 10^{11}$ or $(10^{11})^2$.) 3. Ask students to calculate $(3 \times 2)^3$ two different ways. 4. Simplify: a. $6^7 \div 6^5$ b. $(3^5)^2$ c. $2^3 \times 3^2$ d. -5^2 e. $(\frac{2}{5})^3$ f. $(\frac{2}{3})^{-2} \times (\frac{2}{3})^3$ **Journal** 1. Which is greater, 2^{-4} or 4^{-2}? Explain how you know. (You may use your calculator to confirm your answer.) 2. Solve the following question mentally, and explain your thinking. $3^3 \times 3^{-2} \times 4^3 \times 4^{-3} \times 2^4 \times 2^{-2}$

STRAND: NUMBER (NUMBER OPERATIONS)

GENERAL OUTCOME	Apply exponent laws to solve problems.
SPECIFIC OUTCOME	19. Understand and use the exponent laws to simplify expressions with variables as bases and use substitution to calculate a numerical value. [PS, R, T] (9–9)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Addison-Wesley Mathematics 10*, pp. 52–61
- *Interactions 9*, pp. 30–34, 38–42, 120–123
- *Mathpower 9*, pp. 20–43
- *Minds on Math 9*, pp. 292–299
- *TLE 9*, Laws of Exponents 1–3, Student Refresher pp. 10–15, Teacher's Manual pp. 32–43
- *TLE 9*, Evaluating Powers and Expressions, Student Refresher pp. 16–19, Teacher's Manual pp. 44–51
- *TLE 9*, Simplifying and Evaluating Exponential Expressions, Student Refresher pp. 22–23, Teacher's Manual pp. 50–55

Previously Authorized Resources

- *Journeys in Math 9*, pp. 102–103, 132–133, 138–139
- *Math Matters: Book 2*, pp. 95–101

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Premature use of a calculator in work with exponent laws causes many students trouble. Provide opportunities for students to simplify expressions and to calculate a numerical value without the use of a calculator to ensure that process is understood.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Questions

Questions

1. Mentally, or with paper and pencil, calculate the value of each of the following expressions:
   a. $\dfrac{1+2^2}{3^2-2^2}$
   b. $5 \times 4^6 \div (4^2)^2$
   c. $1 + 7n^2$, if $n = 3$
   d. $(6^2)^3 \div (6^2)^2$
   e. $\dfrac{6^2 \div 2^2 - 2}{5^2 - 14}$

Calculations with powers are done easily with a scientific calculator after students have developed an understanding of the exponent laws and can apply them to simple expressions.

Students often have trouble doing the keying sequence for fractions on the calculator as they omit the brackets; e.g.:

For $\left(\dfrac{4}{5}\right)^3$, students will often input 4 ÷ 5 y^x 3 = which won't give the correct answer.

Sometimes it is easier to simplify within parentheses first; other times it is easier to apply the power law first; e.g.:

Evaluate $\left(\dfrac{6^3}{4^2}\right)^2$

Method 1: $\left(\dfrac{6^3}{4^2}\right)^2 = \dfrac{6^6}{4^4} = \dfrac{46656}{256} = 182.25$

Method 2: $\left(\dfrac{6^3}{4^2}\right)^2 = \left(\dfrac{216}{16}\right)^2 = (13.5)^2 = 182.25$

Students should be familiar with both methods.

2. Evaluate each of the following, using both methods of simplifying. Express the result as a whole number or fraction in lowest terms.
   a. $(4^3)^2$
   b. $\left(\dfrac{9^2}{3^3}\right)^{-2}$
   c. $(4^3 \times 2^{-3})^2$

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Paper and Pencil**
	1. Simplify, then evaluate each expression for $x = -2$. a. $x^3 \bullet x^2$ b. $\frac{x^6}{x^3}$ c. $(x^2)^3$ d. $x^{-3} \bullet x^{-1}$ e. $\frac{x^{-2}}{x^{-3}}$ f. $(x^{-2})^{-1}$
Keying sequences are not unique. Students should be encouraged to explain their own preferred keying sequence.	2. Calculate $(1.6 \times 2.5)^3$ using your calculator. Describe and explain the keying sequence you used.
	3. Evaluate the following, without using a calculator: a. $(5-2)^3 - 7$ b. $\left(\frac{4^2+2}{2}\right)^2$ c. $7^3 - 7^2 \times 2$
	4. Evaluate the following, using a calculator: a. $(3.14 \times 7^2) \times (5.26 \times 4)^2$ b. $\left(\frac{2}{3}\right)^5 \div \left(\frac{2}{3}\right)^3$ c. $\frac{(9^3 \times 8^4)^2}{3^4}$ d. $\left(\left(-\frac{1}{5}\right)^2\right)^3$
	5. Use the rules of powers and exponents to find the value of n. a. $(5^n)^n = 625$ b. $(4^n)^3 = 4^{-6}$ c. $(n^2)^3 = 64$ d. $n^5 \div n^3 = 36$

STRAND: PATTERNS AND RELATIONS (PATTERNS)

GENERAL OUTCOME

Generalize, design and justify mathematical procedures, using appropriate patterns and technology.

SPECIFIC OUTCOME

1. Generalize a pattern arising from a problem solving context using mathematical expressions and equations, and verify by substitution. [C, CN, PS, R] (8–1, 9–1)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Interactions 7*, pp. 206–209
- *Interactions 8*, pp. 30–36, 222–226, 229–230
- *Interactions 9*, p. 96
- *Mathpower 7*, pp. 194–195, 202–205
- *Mathpower 8*, pp. 8–9, 146–151, 154–158
- *Mathpower 9*, pp. 58–66
- *Minds on Math 8*, pp. 350–351
- *TLE 7*, Patterns and Relations, Student Refresher pp. 44–45, Teacher's Manual pp. 100–103
- *TLE 9*, Logic, Problem Solving and Mathematical Modelling, Student Refresher pp. 26–29, Teacher's Manual pp. 64–71

Previously Authorized Resources

- *Journeys in Math 8*, pp. 266–267
- *Journeys in Math 9*, pp. 222–223

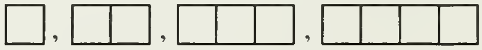
TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Students should be able to look at a sequence of numbers or a diagram and describe the pattern displayed. They should be able to use mathematical expressions and equations to describe the pattern. 1. 6, 10, 14, 18, … a. Describe the pattern in the numbers above. b. Give the next three numbers in the pattern. Solution a. Each number is 4 greater than the previous one. b. 22, 26, 30

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

2. Toothpicks are used to form a series of squares as shown.



a. Copy and complete the following table.

No. of squares	1	2	3	4	5	6	7
No. of toothpicks needed	4	7	10	?	?	?	?

b. Describe in words the relationship between the number of toothpicks needed and the number of squares being formed.

c. Write an equation that shows the relationship between the number of toothpicks needed and the number of squares being formed.

d. Use your equation from part c to determine the number of toothpicks needed to form a row of 20 squares.

Solution

a.

No. of squares	1	2	3	4	5	6	7
No. of toothpicks needed	4	7	10	13	16	19	22

b. The number of toothpicks needed is one greater than three times the number of squares.

c. $t = 3s + 1$

d. $t = 3s + 1$
$t = 3(20) + 1$
$t = 61$
The number of toothpicks needed for 20 squares is 61.

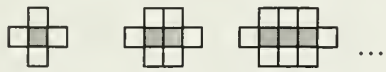
TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

Paper and Pencil[1]

1. For each of the tile patterns:
   a. make a table of values, and describe the pattern in words
   b. use the pattern and description to write a mathematical equation identifying what the variable(s) represent
   c. use the equation to help determine the tenth entry in the table.

   i. 

   ii. 

2. A certain rectangle has a length that is $\frac{1}{2}$ of the width.
   a. Make a table showing the relationship between the width and the perimeter.
   b. Describe, in words, the relationship between the width and the perimeter.
   c. Write a mathematical rule to relate the width and perimeter, identifying what the variable(s) stand for.
   d. Use the rule to find the perimeter when the width is 99 metres.

[1] Paper and Pencil questions 1 and 2 are adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 8.*

STRAND: PATTERNS AND RELATIONS (PATTERNS)

GENERAL OUTCOME Generalize, design and justify mathematical procedures, using appropriate patterns and technology.

SPECIFIC OUTCOME 2. Given a first degree equation, substitute numbers for variables and graph and analyze the relation. [C, PS, R, V] (8–2)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Addison-Wesley Mathematics 10*, pp. 164–167, 186, 188, 223–233
- *Interactions 8*, pp. 227–228, 284–289
- *Mathpower 7*, pp. 212–217
- *Mathpower 8*, pp. 162–167
- *Minds on Math 8*, pp. 118–119, 355
- *TLE 9*, Mathematical Modelling, Student Refresher pp. 28–29, Teacher's Manual pp. 68–71

Previously Authorized Resources
- *Journeys in Math 8*, pp. 310–311
- *Journeys in Math 9*, pp. 310–321
- *Math Matters: Book 2*, p. 224

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	The first-degree equations required in the course are of four types: 1. $y = mx$, with m positive 2. $y = mx + c$, with m a natural number 3. $y = mx + c$, with m a positive rational number 4. $y = mx + c$, with m negative The best procedure is to gather data, usually four or five data pairs, sufficient to determine the linear equation, and then use the equation to predict the value of y for a given value of x and also to determine the required value of x that is needed to yield a given value of y. It is recommended that the variable x be referred to as either the independent variable or the manipulated variable, and that the variable y be referred to as the dependent variable or the responding variable.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Examples Case 1 The independent variable could refer to the hours worked, m could refer to the hourly wage rate, and the dependent variable could refer to the gross wages. In this case, y and x could be replaced by W (wages) and t (time). Case 2 The independent variable could refer to the number of quarters added to a purse, m would equal 25, and the dependent variable could refer to the total money in the purse. In this case, y, x and c could be replaced by F (finishing money in cents), n (number of quarters added) and S (starting money in cents). Case 3 The independent variable could refer to the volume of a liquid in a beaker, m could refer to the density of the liquid, and the dependent variable could refer to the combined mass of the liquid and beaker. In this case, y, x and c could be replaced by M (total mass), V (volume of liquid) and B (mass of the empty beaker). Case 4 The independent variable could refer to the advertised cost of a pair of shoes, m would be fixed at -1.07 to allow for a 7% GST, and the dependent variable could refer to the change given from \$100. In this case, y, x and c could be replaced by C (change), S (cost of a pair of shoes) and 100.

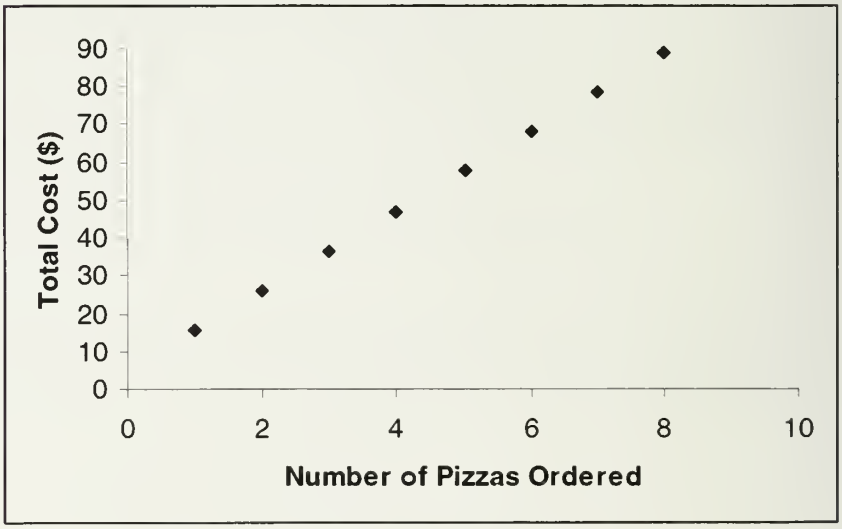
	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	See below

Paper and Pencil

1. Frank's Pizza House charges \$5.00 for delivery and \$10.50 for each small pizza ordered. The total cost can be represented by the equation $C = 5 + 10.5n$, where C is the total cost in dollars, and n is the number of small pizzas ordered.
   a. Use the equation to determine the total cost of seven small pizzas, including delivery.
   b. Use the equation to determine the total cost of four small pizzas, including delivery.
   c. Make a table of values for the cost of pizzas and delivery for one to eight small pizzas. Graph the data.

1. Solution
   a. $C = 5 + 10.5n$
      $C = 5 + 10.5(7)$
      $C = 78.5$
      The total cost is \$78.50.
   b. $C = 5 + 10.5n$
      $C = 5 + 10.5(4)$
      $C = 47$
      The total cost is \$47.00.
   c.

No. of pizzas	1	2	3	4	5	6	7	8
Total cost (\$)	15.50	26.00	36.50	47.00	57.50	68.00	78.50	89.00



STRAND: PATTERNS AND RELATIONS (PATTERNS)

GENERAL OUTCOME Generalize, design and justify mathematical procedures, using appropriate patterns and technology.

SPECIFIC OUTCOME 3. Translate between an oral or written expression and an equivalent algebraic expression. [C, CN] (8–3, 9–8)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Addison-Wesley Mathematics 10*, pp. 156–157, 276–278
- *Interactions 7*, pp. 210–215, 219–220
- *Interactions 8*, pp. 224–226, 239, 284, 286, 298–299
- *Interactions 9*, pp. 224–227
- *Mathpower 7*, pp. 198–199
- *Mathpower 8*, pp. 153, 174–175
- *Mathpower 9*, pp. 66–69
- *Minds on Math 7*, pp. 338–341
- *Minds on Math 8*, pp. 360–365, 472
- *Minds on Math 9*, pp. 139–143, 154–160
- *TLE 8*, Exploring Equations, Student Refresher pp. 42–43, Teacher's Manual pp. 96–99
- *TLE 9*, Mathematical Modelling, Student Refresher pp. 28–29, Teacher's Manual pp. 68–71

Previously Authorized Resources
- *Journeys in Math 8*, pp. 342–343
- *Journeys in Math 9*, pp. 126–127, 174–177
- *Math Matters: Book 2*, pp. 22, 78–79

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	In order to successfully solve problems using algebra, students need to make the connection between the written word and the corresponding algebraic expression. They must be able to identify the pertinent verbal expressions in the problem, and then translate those verbal expressions into algebraic expressions. 1. Write an algebraic equation or expression for the following: a. six more than a number cubed b. five less than a number c. the product of two numbers diminished by five d. four times a number decreased by six e. one quarter of a number is twelve f. six times a number is eighteen

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	1. Solution a. $n^3 + 6$ b. $n - 5$ c. $xy - 5$ d. $4x - 6$ e. $\frac{1}{4}m = 12$ f. $6x = 18$ 2. Write a verbal expression for the following algebraic expressions. a. $3p - 4$ b. $m + 5$ c. $x^2 - 2 = 14$ d. $2t + 6 = 12$ 2. Solution a. four less than three times a number b. five more than a number c. two less than the square of a number is fourteen d. six more than twice a number is twelve 3. Write an equation or equations that could be used to solve the following problems. a. What are the dimensions of a rectangle that is twice as long as it is wide, and whose perimeter is 42 cm? b. Two numbers have a sum of 48. The larger number is four more than the smaller. What are the two numbers? c. Mary is 9 years older than Joan. The sum of their ages is 39. How old is Joan? 3. Solution a. $2(2w + w) = 42$ or $L = 2W, 2L + 2W = 42$ b. $x + x + 4 = 48$ or $L + S = 48, L = S + 4$ c. $J + 9 + J = 39$ or $M = J + 9, M + J = 39$

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Paper and Pencil** 1. An airplane travels eight times as fast as a car. The sum of their speeds is 990 km/h. How fast is each one travelling? 2. John earned four times as much as Jerry. The difference in their earnings is $150.00. How much did Jerry earn? 3. A 23-metre rope is cut into three pieces. The first piece is twice as long as the second piece. The third piece is one metre shorter than the second piece. How long is each piece?

STRAND: PATTERNS AND RELATIONS (PATTERNS)

GENERAL OUTCOME	Generalize, design and justify mathematical procedures, using appropriate patterns and technology.
SPECIFIC OUTCOME	4. Write equivalent forms of algebraic expressions, or equations, with integral coefficients. [C, CN, R] (9–3)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Addison-Wesley Mathematics 10*, pp. 150–155
- *Interactions 9*, pp. 230–232
- *TLE 9*, Equivalent Expressions, Student Refresher pp. 30–31, Teacher's Manual pp. 72–75

Previously Authorized Resources

- *Journeys in Math 9*, pp. 172–173

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes This outcome is crucial for success in Grade 10, as a formula must be rearranged before it can be graphed or inputted into a spreadsheet document.	A formula is often not written in the form needed for a particular problem. For example, the formula for density is $D = \frac{M}{V}$, where M is mass and V is volume. If density and volume are known and mass is unknown, it may be useful to rewrite the formula with mass as the subject: $M = D \times V$. The known values for D and V could then be substituted and the unknown mass calculated. 1. Rewrite each formula to solve for the given variable. a. $A = lw,\ w$ b. $P = 2l + 2w,\ w$ c. $A = \frac{1}{2}bh,\ b$ d. $C = 2\pi r,\ r$ e. $E = mc^2,\ c$

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	1. Solution a. $w = \dfrac{A}{l}$ b. $w = \dfrac{P - 2l}{2}$ c. $b = \dfrac{2A}{h}$ d. $r = \dfrac{C}{2\pi}$ e. $c = \sqrt{\dfrac{E}{m}}$ 2. The distance travelled by a car is given by the formula $d = vt$, where d is the distance in km, v is the speed (velocity) in km/h, and t is the time in hours. Rearrange this formula as needed to solve the following problems. a. How fast would a car have to travel to go 500 km in 4 hours? b. How long would it take a car to go 385 km at 110 km/h? c. How far would a car go in 5 hours at 95 km/h? 2. Solution a. $v = \dfrac{d}{t}$ $v = \dfrac{500}{4}$ $v = 125$ km/h b. $t = \dfrac{d}{v}$ $t = \dfrac{385}{110}$ $t = 3.5$ h c. $d = vt$ $d = (95)\,(5)$ $d = 475$ km

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

Paper and Pencil

1. Complete the table.

Distance	Velocity	Time
760 km		8 h
488 km	80 km/h	
	60 km/h	3 h
225 km		2.5 h
880 km	110 km/h	
	75 km/h	6.5 h

Journal or Interview

1. Explain why each of the following is incorrect.
   a. $3x = 3 + x$
   b. $-x^2 = (-x)^2$
   c. $x - 1 = 1 - x$
   d. $x^2 = 2x$

STRAND: PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

GENERAL OUTCOME

Generalize arithmetic operations from the set of rational numbers to the set of polynomials.

SPECIFIC OUTCOME

5. Identify constant terms, coefficients and variables in polynomial expressions. [C] (9–7)

MANIPULATIVES

- Algebra tiles

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Addison-Wesley Mathematics 10*, pp. 46, 72–75
- *Interactions 7*, pp. 210–211
- *Interactions 9*, pp. 104–105, 127
- *Mathpower 7*, p. 196
- *Mathpower 9*, pp. 16, 62
- *Minds on Math 9*, pp. 320–325
- *TLE 9*, Terms of Polynomials, Student Refresher pp. 42–43, Teacher's Manual pp. 96–99

Previously Authorized Resources

- *Journeys in Math 9*, p. 126
- *Math Matters: Book 2,* pp. 84–86

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Students should be familiar with the terminology but should not be required to memorize formal definitions. Provide opportunities for students to discuss mathematical concepts, and encourage proper usage of terminology in those discussions. Students should be able to give examples of constants, coefficients and variables. A polynomial is made up of several parts. Each of the groups of numbers and/or letters, separated by addition or subtraction signs, is called a term. Each term is made up of a number factor, called the coefficient, and a variable factor (or factors). A term that consists of a number by itself is called a constant. • A monomial consists of one term; e.g., $4n^2$. • A binomial consists of two terms; e.g., $4n^2 + 4mn$. • A trinomial consists of three terms; e.g., $4n^2 + 4mn + 8$.

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

Algebra tiles are cardboard squares and rectangles that are white on one side and coloured on the other. The "x" set consists of a number of unit tiles, (squares that are one unit in length), a number of x tiles (rectangles that are x units long and 1 unit wide), and a number of x^2 tiles (squares that are x units in length). The length of the x tile is not an integral number of 1tiles. Students should be encouraged to make this discovery. There is also a "y" set and an overhead set. The "y" set consists of y tiles (rectangles that are y units long and 1 unit wide), xy tiles (rectangles that are x units wide and y units long), and y^2 tiles (squares that are y units long). Again the y tile is not an integral number of unit tiles. For introductory work, the "x" set is probably adequate.

In the term $4n^2$:

- 4 is the coefficient
- n is the variable.

In the term $4mn$:

- 4 is the coefficient
- m and n are the variables.

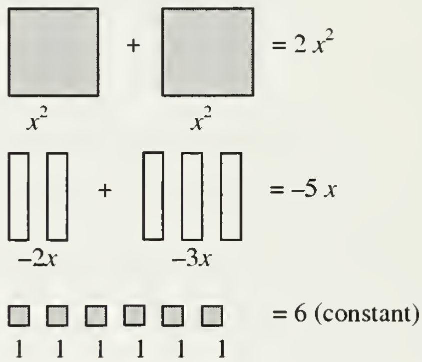
In the term 8:

- 8 is a constant.

Terms with identical variables including their exponents are called like terms. Examples of like terms are $3a$ and $2a$, n^2 and $-4n^2$, and a^2b and $-3a^2b$.

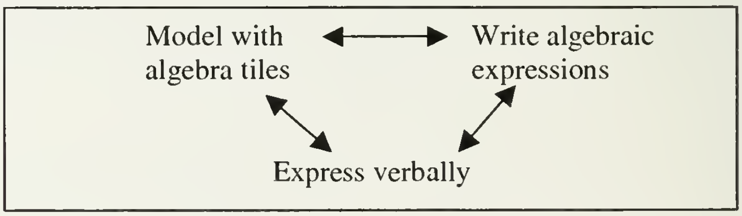
Collecting like terms can be demonstrated concretely using algebra tiles. These tiles can be purchased commercially or can be handmade.

Tiles that are the same shape (like terms) can be combined; e.g.:



Translation of Expressions

To express quantities, using symbols, models and words interchangeably:

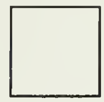


Teaching Notes

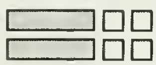
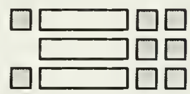
Resources that contain suggestions for the use of algebra tiles can be found in the *Kindergarten to Grade 9 Mathematics Resources: Annotated Bibliography*.

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

The following describes how the algebra tiles used in the activities in this manual will represent the variables and constants shown, unless stated otherwise.

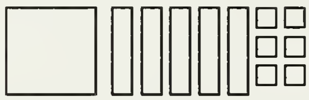
 represents –1
 represents +1
 represents $-x$
 represents $+x$
 represents x^2
 represents $-x^2$

Examples

1. Write an algebraic expression for the quantities illustrated by the algebra tiles.
   a. 
   b. 

2. Arrange algebra tiles to illustrate these algebraic expressions.
   a. $2x + 3$
   b. $-x - 2$

3. Express the following mathematical expressions verbally.
   a. xy
   b. $2x + 1$
   c. $-2(y + 3)$

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Paper and Pencil** 1. What is the coefficient of $-6a^4b$? of $8a^2b^5$? 2. What are the constant terms in the expressions $4x - 3 + 2y$? $7x + 8y - 8$? 3. Write a trinomial involving two variables, having coefficients of 6 and –3, and a constant term of 2. 4. Classify each of the following according to the number of terms. a. $x^2 + 2$ b. $x^2 + 2x + 5y$ c. $3x^2$ d. $xy + 7$ **Performance** 1. Given:  a. What expression does this model represent? b. Identify the constant. c. Identify the coefficient of the: • x term • x^2 term. **Journal/Interview** 1. State the difference between each item in the pair, and give an example of each: • a variable and a constant • a variable and a coefficient • like and unlike terms.

STRAND: PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

GENERAL OUTCOME

Generalize arithmetic operations from the set of rational numbers to the set of polynomials.

SPECIFIC OUTCOME

6. Evaluate polynomial expressions, given the values of the variables. [E] (9–8)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Interactions 7*, pp. 216–217
- *Interactions 9*, pp. 106–109
- *Mathpower 7*, pp. 196–197
- *Mathpower 8*, pp. 152–153
- *Mathpower 9*, pp. 62–65
- *Minds on Math 7*, pp. 355, 359
- *Minds on Math 8*, pp. 352–360, 366–368
- *Minds on Math 9*, pp. 330, 331, 337, 342
- *TLE 9*, Evaluating Polynomials, Student Refresher pp. 44–45, Teacher's Manual pp. 100–103

Previously Authorized Resources

- *Journeys in Math 8*, pp. 344–345
- *Journeys in Math 9*, pp. 128–129, 136, 143
- *Math Matters: Book 2*, pp. 87–88, 92–94, 112–113, 136–137

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Substituting values for the variables of an algebraic expression or formula is a useful skill. It is important that students learn and use proper substitution techniques, such as the use of parentheses and the order of operations. 1. Evaluate the expression, if $x = 3$ and $y = -1$. a. $x^2 - y^2$ b. $-3x + y$ c. $15 - xy$ d. $\frac{-x}{4y}$ e. x^y f. $\frac{x}{5} + \frac{y}{5}$

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

Solution

a. $x^2 - y^2 = (3)^2 - (^-1)^2$
$= 9 - 1$
$= 8$

b. $-3x + y = -3\ (3) + (^-1)$
$= -9 - 1$
$= -10$

c. $15 - xy = 15 - (3)\ (^-1)$
$= 15 - (^-3)$
$= 18$

d. $\dfrac{-x}{4y} = \dfrac{-(3)}{4(-1)}$
$= \dfrac{-3}{-4}$
$= \dfrac{3}{4}$

e. $x^y = (3)^{-1}$
$= \dfrac{1}{3}$

f. $\dfrac{x}{5} + \dfrac{y}{5} = \dfrac{3}{5} + \dfrac{-1}{5}$
$= \dfrac{2}{5}$

2. Complete the following:

x	$x^3 - 1$
2	
–3	
½	
6	
1	

Solution:

x	$x^3 - 1$
2	7
–3	–28
½	$-\frac{7}{8}$
6	215
1	0

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	3. Use the formula $A = \frac{h(a+b)}{2}$ to calculate the area of a trapezoid with the following measurements: base a = 22 cm; base b = 15 cm; and height h = 6 cm. Solution $A = \frac{h(a+b)}{2}$ $A = \frac{6(22+15)}{2}$ $A = 111$ The area of the trapezoid is 111 cm^2.

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

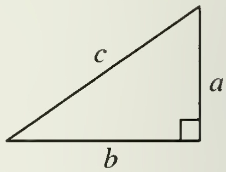
Paper and Pencil

1. Complete the following tables of values.

x	$3(x-2)$
7	
–3	
1	
–2	

x	$2x(3-x)$
1	
3	
–3	
4	

2. The formula for the length of the hypotenuse on the right triangle shown is $c = \sqrt{a^2 + b^2}$. Use proper substitution techniques to determine the length of the hypotenuse if the other two sides are 11.4 cm and 15.2 cm.



3. The formula for the surface area of a soup can is $A = 2\pi r\,(r + h)$, where r is the radius and h is the height of the can. Use proper substitution techniques to determine the surface area of a soup can that has a radius of 4.2 cm and a height of 10 cm.

4. Verify the following equations, by substituting 4 for x and –3 for y.

   a. $-2\,(x - y) = -2x + 2y$
   b. $2x\,(3x - 5y) = 6x^2 - 10xy$
   c. $(x - 3y)\,(x + 3y) = x^2 - 9y^2$

STRAND: PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

GENERAL OUTCOME

Generalize arithmetic operations from the set of rational numbers to the set of polynomials.

SPECIFIC OUTCOMES

7. Represent and justify the addition and subtraction of polynomial expressions, using concrete materials and diagrams. [C, R, V] (9–9)
8. Perform the operations of addition and subtraction on polynomial expressions. [R] (9–10)

MANIPULATIVES

- Algebra tiles

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Addison-Wesley Mathematics 10*, pp. 72–75, 79–81
- *Interactions 9*, pp. 112–117
- *Mathpower 7*, pp. 230–231
- *Mathpower 8*, pp. 178–179
- *Mathpower 9*, pp. 72–73, 146, 150–155
- *Minds on Math 9*, pp. 327–331
- *TLE 9*, Algebra Tiles Explorer
- *TLE 9*, Adding and Subtracting Polynomials with Tiles, Student Refresher pp. 46–49, Teacher's Manual pp. 104–111

Previously Authorized Resources
- *Journeys in Math 9*, pp. 128–131
- *Math Matters: Book 2*, pp. 89–91

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes Refer to Portfolio Question 2 in the Tasks for Instruction and/or Assessment section.	Magic number problems and calendar problems are both fun ways to practise the addition and subtraction of polynomials. Using manipulatives, such as algebra tiles, to introduce polynomials and their operations is useful for most students, particularly those who are still at the concrete stage of developing a conceptual understanding. When first using algebra tiles, give students time to become familiar with them. The coloured sides represent positive values, and the white sides represent negative values. Algebra tiles are particularly useful for operations with polynomials. Adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 9 Implementation Draft.*

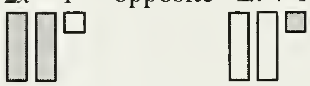
INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

While addition of polynomials is often straightforward, consideration should be given to the different representations for subtraction, including the following:

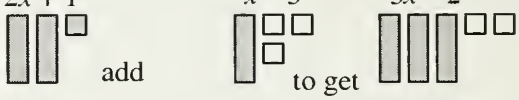
- **comparison**—refers to comparing and finding the difference between two quantities
- **zero pairs**—think of two types of zero pairs
  : $0 = -1 + 1$
  : $0 = -x + x$
- **taking away**—simply refers to starting with a quantity and removing or taking away a specified amount to arrive at an answer. For example, for $(x^2 + 2x - 2) - (x^2 + x + 1)$, if we start with and add a zero pair represented by , then take away , the result is
- **adding the opposite**—refers to subtracting by first changing the question to an addition and then adding the opposite of a quantity. For example, instead of subtracting x, one might add $-x$. Likewise, instead of subtracting $2x - 1$, one might add $-(2x - 1)$, which is the same as $-2x + 1$. Students should model $2x - 1$ and understand that the opposite is found by flipping the tiles.

  $2x - 1$ opposite $-2x + 1$



- **missing addend**—asks the question, "What would be added to the number being subtracted to get the starting amount?" For example, for $(3x - 2) - (2x + 1)$, ask: "What is added to $2x + 1$ to get $3x - 2$?"

  $2x + 1$ $x - 3$ $3x - 2$



Perimeter is a very useful application of addition and subtraction of polynomials.

Adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 8.*

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

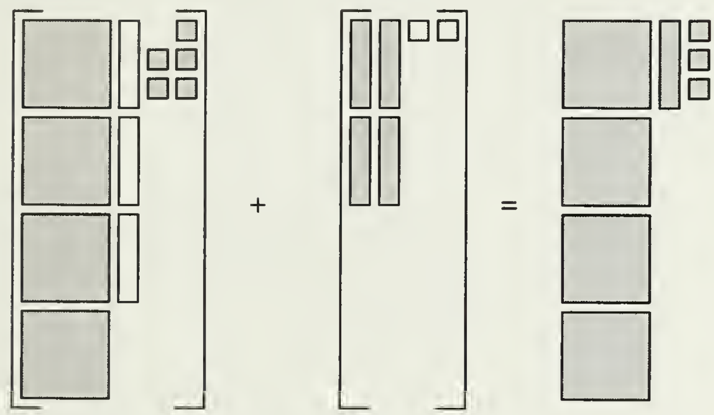
Teaching Notes

Simplifying Expressions

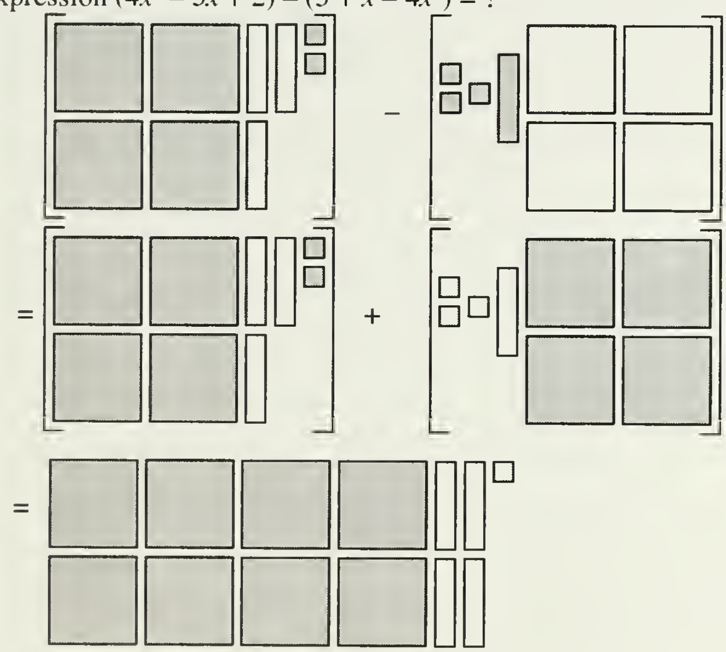
- **use algebra tiles to illustrate the combining of polynomials**

Examples

1. Demonstrate how the algebra tiles shown below are used to simplify the expression $(4x^2 - 3x + 5) + (4x - 2) = ?$



2. Explain how the algebra tiles shown below are used to simplify the expression $(4x^2 - 3x + 2) - (3 + x - 4x^2) = ?$



	INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

3. Explain how algebra tiles can be used to illustrate the algebraic process for simplifying the following.
   a. $(2x^2 + 3x + 2) + (x^2 - 5x - 1)$
   b. $(4x^2 - 2x - 3) - (2x^2 - 3x)$

- **recognize and combine like terms**

Stress to students that the sign in front of any term must go with it.

Right	Wrong
$a - b + c$	$a - b + c$
$a + c - b$	$a - c + b$

Rearranging terms makes no difference to the result. This can be demonstrated easily with algebra tiles. However, discuss with students how reordering terms can aid in mental mathematics.

Examples

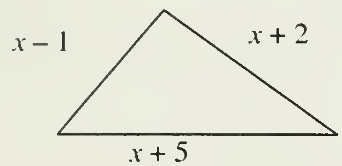
1. Which of these have the same value as 38 + 46 – 13 – 8 + 25?
   a. 38 – 13 + 46 + 25 – 8
   b. 38 + 25 + 46 – 13 – 8
   c. 38 – 46 + 13 – 8 + 25

2. Which of these expressions are equivalent to $a - b + c - d$?
   a. $a + c - b - d$
   b. $a - d - b + c$
   c. $a + c - d + b$
   d. $a + d - b + c$

3. The terms $9ab$ and $-9ab$ are like terms. What single change could you make to one of the terms so that they would be unlike terms? State at least three possible changes.

4. Combine like terms.
   a. $7x - 5x + x$
   b. $2b - 3 - 5b + 1$
   c. $-x - 3y + 6x + y$
   d. $-3xy + 5xz - 4xy - 4xz$

5. Simplify:
   a. $5x - 7x + 2x$
   b. $-5xy + 3xy$
   c. $(3x - 8) - (x^2 + 5x - 6)$
   d. $(5x^3 + 3m + 2) + (-2x^3 + 5p - 6)$
   e. Subtract $(-2x^2 + 5x - 3)$ from $(5x^2 - 3x + 7)$

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	6. The perimeter of a quadrilateral is $(10x + 4y + 16)$. What is a possible expression for the length of each side? 7. Simplify $3a - 8b + 7a - 15b + 10$. 8. C represents the number of compact discs, and $C + C + 4 + 2C = 56$. Using this information, write a problem. 9. Write an expression for the perimeter of the triangle below.  • **remove parentheses, and combine like terms** Examples 1. $-(3x - y) = ?$ 2. $7 - (p - 1) - (1 - p) = ?$ 3. $(b - 3a) + (1 - 2b) - (2a + 5) = ?$ 4. Subtract $(-2x + 2)$ from $(2x - 7)$. 5. $-(5 - 6x) - [-(6 - 5x - 2)] = ?$ 6. $-2(2y + 1) - [-(2y - 1)] = ?$ 7. One expression has been subtracted from another. What might the expressions be, if the difference is $-3x^2 - 4$?

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	

Journal

1. Describe to a student who was absent for today's lesson how to use algebra tiles to add polynomials.

2. Describe how you feel about using algebra tiles to help you learn mathematics.

Performance

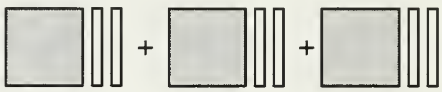
1. Using algebra tiles, illustrate two binomials whose sum is $-x + 2$. Give three solutions.

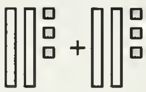
2. What binomial is missing? Tiles may be used.
   $(3x + 1) + (\quad\quad) = 2x + 3$

3. Ask students to show, through the use of algebra tiles, how the solutions to the following differ from each other.
   a. $(2x^2 + x) + (-4x^2 + 5x)$
   b. $(2x^2 + x) - (-4x^2 + 5x)$

Paper and Pencil[1]

1. Ask students to record, symbolically, an expression for each of the following.

   a. 

   b. 

2. Ask students to write the dimensions and area for the rectangle shown.

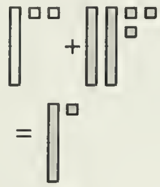
[1] Paper and Pencil questions 1 to 4 are adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 8*. Questions 11 to 13 are adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 9 Implementation Draft*.

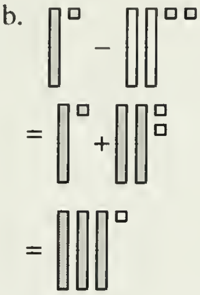
Strand: Patterns and Relations (Variables and Equations)
Specific Outcomes: 7. Represent and justify the addition and subtraction of polynomial expressions, using concrete materials and diagrams. [C, R, V] (9–9)
8. Perform the operations of addition and subtraction on polynomial expressions. [R] (9–10)

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT

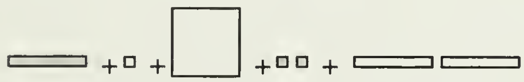
Teaching Notes

3. Write each of the steps in the problems that follow, using symbols, and explain each step.

a. 

b. 

4. Simplify the following expression.



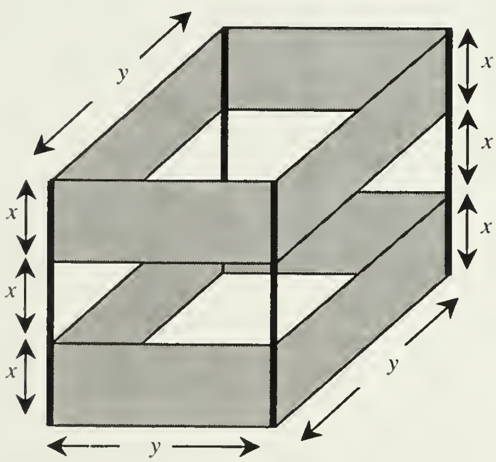
5. State three algebraic examples of two binomials whose difference is $2x - 3$.

6. Find k, if $-3x + 5x + kx = 7x$.

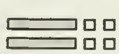
7. Find the value of k and t, if $3x + 2y - x + 4y = kx + ty$.

8. Box kites are made from lengths of wire, with fabric wrapped around them.

Write an expression for the length of wire needed for the kite shown below.



9. Write an algebraic expression for the quantity illustrated by the algebra tiles.

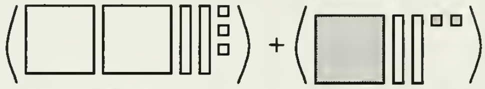
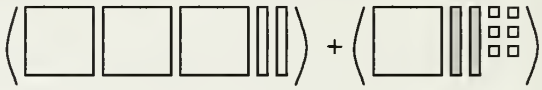
a. 

b. 

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

10. Write an algebraic expression for the quantity illustrated by the algebra tiles, if:
    represents $+y$
    represents $-y$
    represents $+1$
    represents -1
    a. 
    b. 

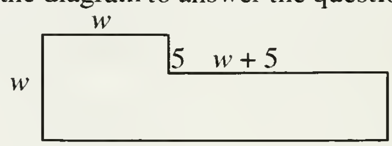
11. Write an algebraic expression for each of the following, and simplify.
    a. 
    b. 
    c. 

12. Sketch models to illustrate each algebraic expression, and use the models to simplify.
    a. $(2x^2 - 5x) - (-3x^2 + 2x)$
    b. $(-3y^2 - 2xy) + (y^2 + 4xy)$

13. List three different pairs of polynomials whose:
    a. sum is $3w^2 - 5w + 4$
    b. difference is $3w^2 - 5w + 4$.

Portfolio[1]

1. Use the diagram to answer the questions below.



   a. Find a polynomial expression that represents the perimeter.
   b. If $w = 8$, find the perimeter using two forms of the polynomial expression. Which calculation was easier? Why?

[1] Portfolio questions 1 and 2 are reproduced with permission from *Atlantic Canada Mathematics Curriculum: Grade 9 Implementation Draft.*

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	2. The following array represents a calendar for September. 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 Note that when any two-by-two array is selected from this calendar that the sum of the diagonal numbers is always the same. For example: 12 13 12 + 20 = 19 + 13 19 20 a. If we let x equal the first number in the two-by-two array, what will the number to the right and the number below it equal? b. Express the sum of the diagonal numbers algebraically.

STRAND: PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

GENERAL OUTCOME

Generalize arithmetic operations from the set of rational numbers to the set of polynomials.

SPECIFIC OUTCOME

9. Represent multiplication, division and factoring of monomials, binomials and trinomials of the form $x^2 + bx + c$ using concrete materials and diagrams. [R, V] (9–11)

MANIPULATIVES

- Graph paper
- Algebra tiles
- Base-ten blocks

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Interactions 9*, pp. 118–129
- *Mathpower 9*, pp. 158–161, 188–203
- *Minds on Math 9*, pp. 338–345, 350–353, 358–365
- *TLE 9*, Binomial Grid Explorer
- *TLE 9*, Factoring Polynomials with Tiles, Student Refresher pp. 50–51, Teacher's Manual pp. 112–115

Previously Authorized Resources
- *Journeys in Math 9*, pp. 132–135, 138–147

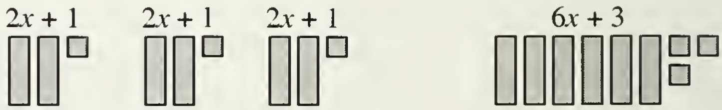
TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	See below

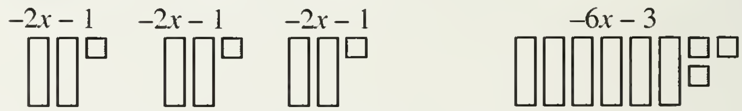
Multiplication of a Polynomial by a Constant

Algebra tiles can be used by students to do a variety of questions; e.g., $3(x + 2)$—use three groups of $x + 2$, and simplify the result.

Multiplication of a polynomial by a constant should be developed with concrete materials and diagrams, using repeated addition. Given a problem such as $3(2x + 1)$, students should recognize that it is the same as $2x + 1 + 2x + 1 + 2x + 1$ and, therefore, model the binomial three times, combine the like terms and arrive at an answer, as shown below.

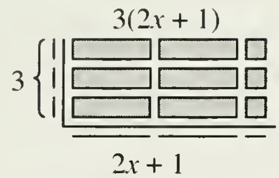


Multiplication of a negative constant; e.g., $-3(2x + 1)$:



Adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 8.*

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	The area model should also be explored in association with the topic, so that students can relate results achieved through repeated addition with results achieved using the area model.



Reproduced with permission from *Atlantic Canada Mathematics Curriculum: Grade 8.*

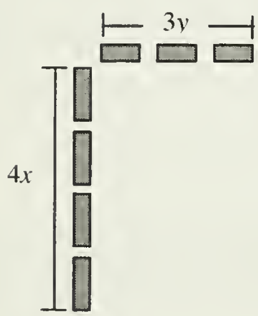
Multiplication of a Monomial by a Monomial

One method of illustrating multiplication of polynomials is through the use of area models. Base-ten blocks or algebra tiles are very helpful.

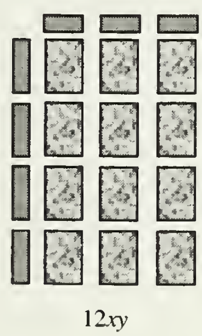
Example

1. Use algebra tiles and an area model to explain the multiplication $(4x)(3y)$.
   a. Set up the model by drawing a frame with dimensions $4x$ and $3y$.
   b. Show how to fill the area model in to get the product.

a.



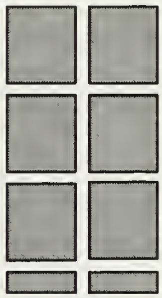
b.



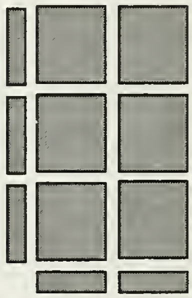
	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	

Division of a Monomial by a Monomial

1. Use algebra tiles and an area model to explain the division $\frac{6x^2}{2x}$.
   a. Set up an area model, using six x^2 tiles, with $2x$ as one of the dimensions.
   b. Identify the other dimension by completing the frame. This will give the solution to the division question.

a. 

$(2x)$

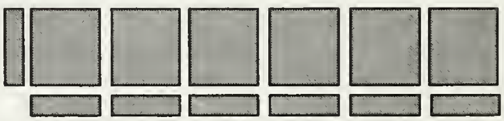
b. 

The solution to $\frac{6x^2}{2x}$ is $3x$.
Thus, the factors of $6x^2$ are $(3x)$ and $(2x)$.

Challenge: Find another set of factors for $6x^2$, using algebra tiles.

Solution:

The dimensions/factors could be $6x$ and x, as shown in the diagram below.



	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	

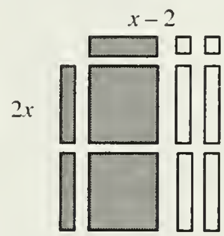
Multiplication and Division of a Polynomial by a Monomial

Multiplication of a Polynomial by a Monomial

Similarly, multiplication of a polynomial by a monomial can be demonstrated concretely with area models.

Example

1. Explain why the area model with algebra tiles can justify the product $2x(x - 2) = 2x^2 - 4x$.



Division of Monomials and Polynomials by Monomials

You determined the width of the rectangle by dividing the area by the known dimension. Similarly, you can divide a polynomial by a monomial that may or may not contain a variable.

Examples

1. Divide $6x^2$ by 3.

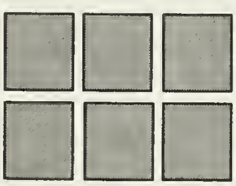
   Solution

   Divide the tiles into three equal groups. There are two x^2 tiles in each group. Therefore, $6x^2 \div 3 = 2x^2$.

2. Divide $6x^2$ by $3x$.

   Solution

   Use tiles to represent $6x^2$.



The width is $2x$,

so $\dfrac{6x^2}{3x} = 2x$.

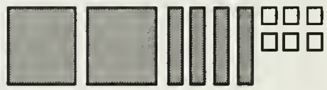
INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

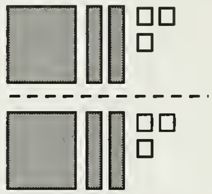
3. Divide $(2x^2 + 4x - 6)$ by 2.

Solution

Use tiles to represent $2x^2 + 4x - 6$



Divide the tiles into two equal groups.



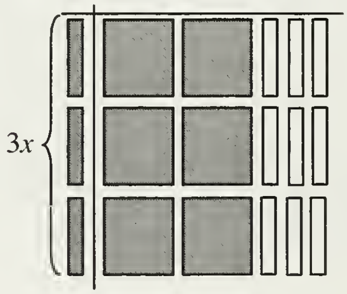
Count the number of tiles of each type there are in each group.

There is one x^2 tile, two x tiles and three negative unit tiles in each group. Therefore, $(2x^2 + 4x - 6) \div 2 = x^2 + 2x - 3$.

4. Divide $6x^2 - 9x$ by $3x$.

Solution

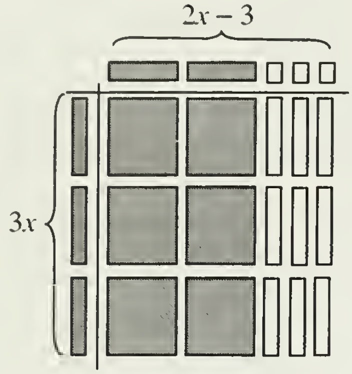
Use algebra tiles to create an area model of a rectangle with an area of $6x^2 - 9x$ and a width of $3x$.



$A = 6x^2 - 9x$

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Now use the area model to find the length of the rectangle.



$A = 6x^2 - 9x$

The length of the rectangle is $2x - 3$.
Therefore, $(6x^2 - 9x) \div 3x = 2x - 3$.

Once several rectangles are constructed, students can look at the dimensions of a rectangle and relate the dimensions to the factors. They will then come to understand that the factors are the dimensions of a rectangle, whereas the area is the product of the factors. They can then be given the dimensions (factors) and asked to find the area (product). The product of $(x + 2)(x + 3)$ is the area of a rectangle that has a length of $x + 3$ and a width of $x + 2$. Students should understand that the x tile has an area of x square units but is also x units in length and 1 unit in width.

To model, the rectangle should be subdivided into four regions. Each region is filled with particular tiles according to the following scheme:

region 1: x^2 tiles only
region 2: horizontal x tiles
region 3: vertical x tiles
region 4: rectangular array of unit tiles

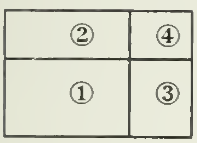
Adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 9 Implementation Draft.*

Multiplication of a Binomial by a Binomial

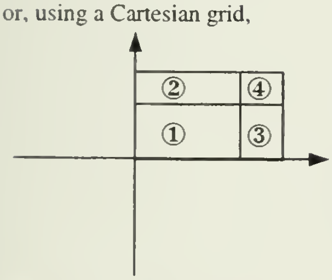
For multiplication of binomials, start by giving students a collection of algebra tiles, such as $x^2 + 6x + 5$, and ask them to create a rectangle by placing the unit tiles at the corner of the x^2 tiles and the x tiles along the sides of the x^2 tiles. When they create the rectangle they should spend some time reflecting on whether it is the only possible rectangle for the given materials. Students should record the dimensions of the rectangle. They should repeat this process several times with several trinomials until they are used to how to place the tiles. Then reverse the process, giving students the dimensions for a rectangle and asking

Teaching Notes

Either



or, using a Cartesian grid,



with region 4 at any corner of region 1, depending on the signs of b and c.

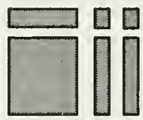
	INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

them to fill in the area and then record the value of that area. Since students should already know that $l \times w = A$, they can record their findings as a product and begin establishing the product and factors relationship. By comparing the symbols representing dimensions with those representing area, using several examples of both cases, and by recording their observations, students should establish a pattern.

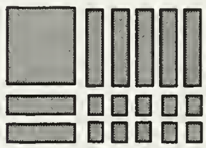
There should be a systematic order for introducing the trinomials. The following order is suggested.

1. c positive and prime, b positive



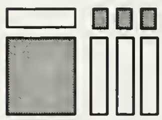
$x^2 + 3x + 2 = (x + 2)(x + 1)$

2. c positive and composite, b positive

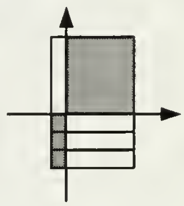


$x^2 + 7x + 10 = (x + 5)(x + 2)$

3. c positive and prime, b negative

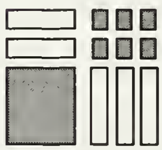


or

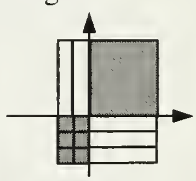


$x^2 - 4x + 3 = (x - 1)(x - 3)$

4. c positive and composite, b negative



or



$x^2 - 5x + 6 = (x - 3)(x - 2)$

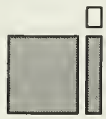
	INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

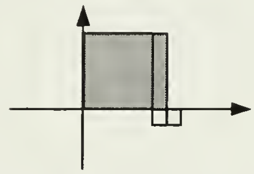
5. c negative and prime, b anything

Here zero pairs have to be added to complete the picture.

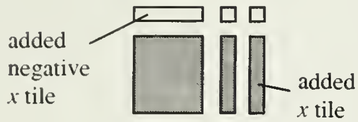
$x^2 + x - 2$



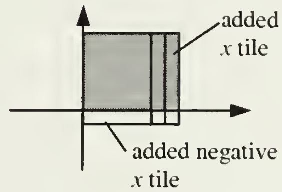
or



Now add one x tile and one negative x tile to complete the area.



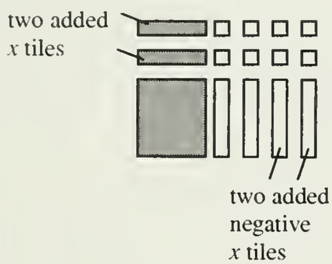
or



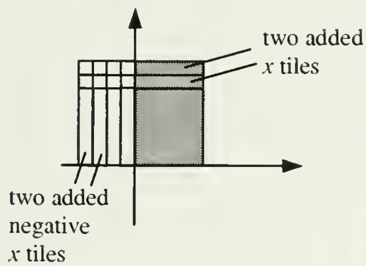
$x^2 + x - 2 = (x + 2)(x - 1)$

6. c negative and composite, b anything

$x^2 - 2x - 8$



or



$x^2 - 2x - 8 = (x - 4)(x + 2)$

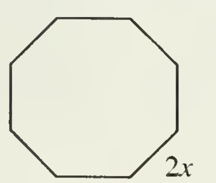
Tables can also be used for multiplication of polynomials; e.g.: $(a + 3)(a - 9)$

	a	$+3$
a	a^2	$3a$
-9	$-9a$	-27

$(a + 3)(a - 9) = a^2 - 6a - 27$

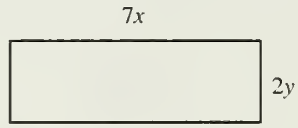
Sample Questions

1. Express the perimeter of the regular octagon in terms of x.

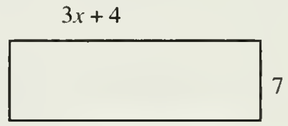


	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	2. Use a rectangular area model:

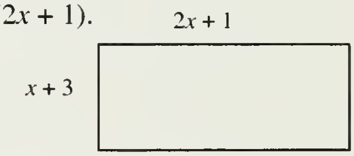
a. with dimensions $7x$ and $2y$ to find the product $(7x)(2y)$



b. with dimensions $3x + 4$ and 7 to find the product $(7)\ (3x + 4)$



c. with dimensions $(x + 3)$ and $(2x + 1)$ to find the product $(x + 3)(2x + 1)$.

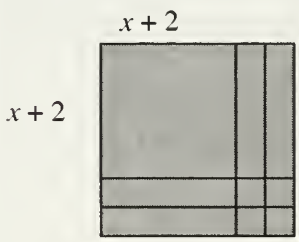


3. Use algebra tiles to represent each of the following products.
   a. $2x(x + 3)$
   b. $3(2x + 1)$
   c. $(x + 2)(2x + 1)$
   d. $(2x + 1)(x - 1)$

4. Use algebra tiles to represent the factoring of the following polynomials.
   a. $6x + 9$
   b. $2x^2 + 6x$
   c. $x^2 + 8x + 12$
   d. $x^2 + 4x + 3$

5. Use algebra tiles to represent the following division.
   $(6x^2 + 8x) \div (2x)$

6. Natalia modelled the process of factoring $x^2 + 4x + 4$, by using algebra tiles and forming a square with them.



What are the factors of $x^2 + 4x + 4$?

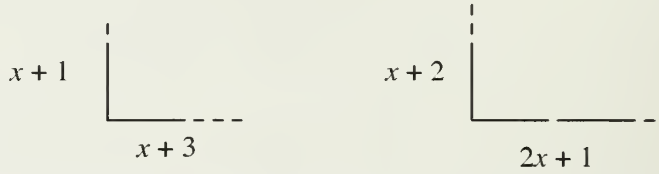
Use Natalia's method to factor $x^2 + 5x + 6$.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	7. Ask students to write, symbolically, the dimensions and area for the rectangle shown.  8. Ask students to present the product for each of the following, using algebra tiles or diagrams. a. $2(x^2 + 3)$ b. $3(2x - 1)$ c. $3(x^2 - 2x + 1)$ 9. Ask students to show the product of 3 and $2x + 4$ as the area of a rectangle.

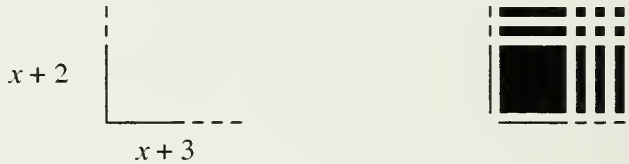
	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Performance**

Performance

1. Create the product rectangle, using algebra tiles, and record the factors and the product symbolically.

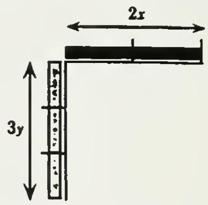


2. In the first diagram, create a rectangle for the dimensions shown; and in the second, record the dimensions for the given area. Compare and discuss the results.



3. Represent the binomial $12x - 6$ with area models, using algebra tiles, in three different ways. Using the models, write $12x - 6$ as the product of a constant and a binomial.

4. a. Justin used algebra tiles and an area model to explain the multiplication $(2x)(3y)$. He set up the model by drawing a frame with dimensions $2x$ and $3y$.



   Show how he filled in the area model to get the product.

   b. Use an area model with algebra tiles to find and justify the product $2(x - 2)$.

5. Form $x^2 + 5x$, using algebra tiles.
   a. Create a rectangle where x is one of the dimensions.
   b. What is the other dimension?
   c. Write a division sentence for the situation.

Portfolio/Journal[1]

1. Show how the process of multiplying 3×2 is similar to the process of multiplying $(x + 1)(x + 3)$, by using area models.

[1] Portfolio/Journal questions 2 and 3 are adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 9 Implementation Draft.*

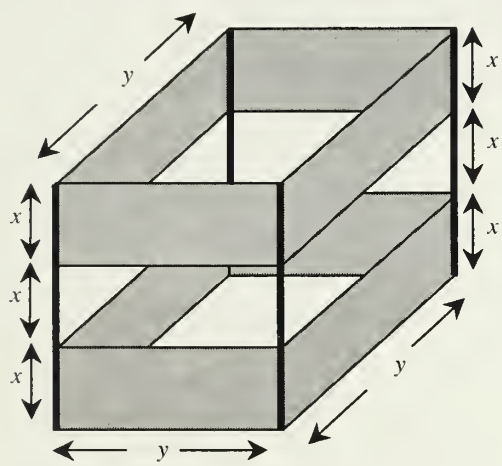
	TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

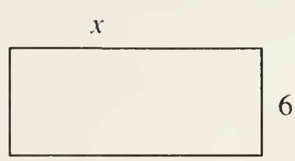
2. Examine the pattern of the factors for each of the following:
   $x^2 + 2x + 1$, $x^2 + 4x + 4$, $x^2 + 6x + 9$, $x^2 + 8x + 16$.
   a. Explain the pattern that you observe.
   b. Create at least two other polynomials whose factors are consistent with this pattern.
   c. Can you think of a concise way of writing the factors?
   d. Use what you have learned in parts a–c to find each of the following:
   $(x + 5)^2$, $(x + 6)^2$, $(x + a)^2$, $(x + 2b)^2$.

3. Examine the pattern of the factors for each of the following:
   $x^2 + 3x + 2$, $x^2 + 4x + 3$, $x^2 + 5x + 4$.
   a. Explain the pattern that you observe.
   b. Create at least two other polynomials whose factors will be consistent with this pattern.
   c. Use the pattern you have observed to find each of the following products quickly:
   $(x + 1)(x + 5)$, $(x + 1)(x + 9)$, $(x + 1)(x + 50)$.

4. Box kites are made from lengths of wire, with fabric wrapped around them. Using the diagram of the box kite, write an expression for the amount of fabric used.



5. The width of a rectangle is 6 and the length is x. If the length of the rectangle is increased by 3, by how much does the area increase?



STRAND: PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

GENERAL OUTCOME

Generalize arithmetic operations from the set of rational numbers to the set of polynomials.

SPECIFIC OUTCOME

10. Find the product of:
    - two monomials
    - a monomial and a polynomial
    - two binomials.

    [R] (9–12)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Addison-Wesley Mathematics 10*, pp. 76, 78, 84–91
- *Interactions 9*, pp. 118–120, 150–152, 159–161
- *Mathpower 9*, pp. 158–161, 188–189, 193–194, 198–199, 202–203
- *Minds on Math 9*, pp. 334–342, 350–355
- *TLE 9*, Multiplying Polynomials, Student Refresher pp. 52–53, Teacher's Manual pp. 116–119

Previously Authorized Resources
- *Journeys in Math 9*, pp. 132–135, 142–143
- *Math Matters: Book 2*, pp. 106–113, 118–128

TECHNOLOGY CONNECTIONS

<table>
<tr><th></th><th>INSTRUCTIONAL STRATEGIES/SUGGESTIONS</th></tr>
<tr><td>Teaching Notes</td><td>Before teaching multiplication of polynomials, it may be useful to review the following concepts with students:
• polynomial vocabulary—coefficient, variable, monomial, binomial
• multiplication of integers
• addition of integers
• exponent laws
• distributive property.

To multiply two monomials, multiply their coefficients and multiply their variables.
$2x^2y(-4xy) = -8x^3y^2$

To multiply a monomial and a polynomial, use the distributive property.
$2x(5x^2 - 3x + 1) = 10x^3 - 6x^2 + 2x$</td></tr>
</table>

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	To multiply two binomials, use the distributive property twice, then simplify. $(x-3)(2x+1) = 2x^2 + x - 6x - 3$ $= 2x^2 - 5x - 3$ 1. Multiply. a. $(-3x)(4xy)$ b. $(5a^2b)(2a^3b)$ c. $(-7m)(4)$ d. $(-xy)(-x^3y^4)$ e. $(8d^3ef)(-ef^2g)$ Solution a. $-12x^2y$ b. $10a^5b^2$ c. $-28m$ d. x^4y^5 e. $-8d^3e^2f^3g$ 2. Multiply and simplify. a. $2x(5-3x+x^2)$ b. $-5y(-y-8)$ c. $(4xy^3-x^2+9)8y$ d. $7(-10p-q^3-6)$ e. $-2x(x+5)+3x(x-1)$ f. $4x(y+2)-(y-1)$ Solution a. $10x-6x^2+2x^3$ b. $5y^2+40y$ c. $32xy^4-8x^2y+72y$ d. $-70p-7q^3-42$ e. $-2x^2-10x+3x^2-3x = x^2-13x$ f. $4xy+8x-y+1$ 3. Expand and simplify. a. $(x+5)(x-4)$ b. $(x-3)(x-2)$ c. $(x+6)(x+4)$ d. $(-2x+y)(3x-4)$ e. $(3-2m)(4+6m)$

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Solution a. $x^2 - 4x + 5x - 20 = x^2 + x - 20$ b. $x^2 - 2x - 3x + 6 = x^2 - 5x + 6$ c. $x^2 + 4x + 6x + 24 = x^2 + 10x + 24$ d. $-6x^2 + 8x + 3xy - 4y$ e. $12 + 18m - 8m - 12m^2 = 12 + 10m - 12m^2$

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT

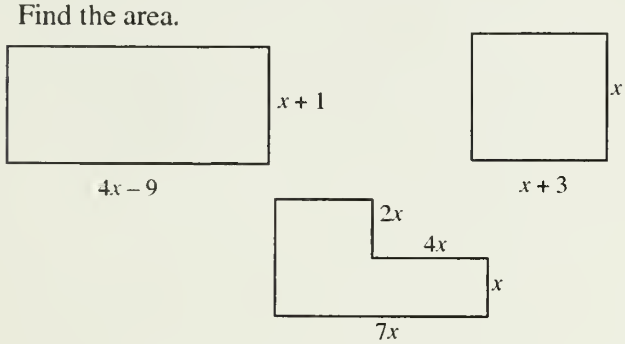
Teaching Notes

Paper and Pencil

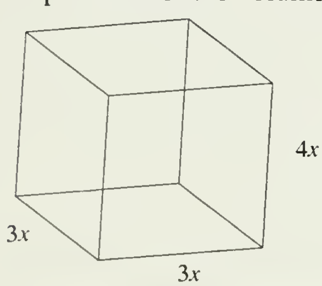
1. Complete the table.

Factor	Factor	Product
(x^3)	$2x + 9$	
$-2ab$	$-6a$	
$x - 1$	$2x + 3$	
$-5a^2bc$	$2abc$	
$-4x$	$x^3 - x^2 + 2$	
$2\ell^2m^2n^2$	$3\ell^2mn^4$	
$(x - 6)$	$(x - 4)$	
$8 - m$	$7 + m$	

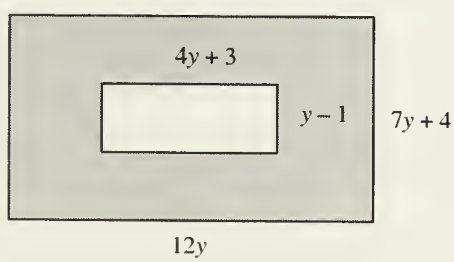
2. Find the area.



3. Write an expression for the volume. Find the volume.



4. Write an expression for the shaded area.



STRAND: PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

GENERAL OUTCOME

Generalize arithmetic operations from the set of rational numbers to the set of polynomials.

SPECIFIC OUTCOME

11. Determine equivalent forms of algebraic expressions by identifying common factors. [PS, R] (9–13)

MANIPULATIVES

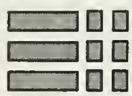
SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Addison-Wesley Mathematics 10*, pp. 93–96
- *Interactions 9*, pp. 154–158
- *Mathpower 9*, pp. 178, 182–186
- *Minds on Math 9*, pp. 343–347
- *TLE 9*, Dividing Polynomials by Monomials, Student Refresher pp. 54–55, Teacher's Manual pp. 120–123

Previously Authorized Resources
- *Journeys in Math 9*, pp. 144–145
- *Math Matters: Book 2*, pp. 121–123, 134

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Before teaching common factoring, it may be useful to review the following concepts with students: • greatest common factors of numbers • prime factorization of numbers • divisibility rules • division of monomials by monomials. Students should understand that factoring is the reverse of multiplication—what multiplication does, factoring undoes. Multiplication — Factoring $2 \times 3 = 6$ — $6 = 2 \times 3$ $(5x)(2x) = 10x^2$ — $10x^2 = (5x)(2x)$ $2(x + 3) = 2x + 6$ — $2x + 6 = 2(x + 3)$

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	It may be advisable to begin with something students have done before—the prime factorization method of determining the greatest common factor (GCF) of whole numbers. $\left.\begin{array}{l}15 = 3\bullet 5\\ 9 = 3\bullet 3\end{array}\right\}\text{GCF} = 3$ $\left.\begin{array}{l}18 = 3\bullet 3\bullet 2\\ 12 = 3\bullet 2\bullet 2\end{array}\right\}\text{GCF} = 3\bullet 2 = 6$ Extend the above concept to monomials. $\left.\begin{array}{l}3m = 3\bullet m\\ 6 = 3\bullet 2\end{array}\right\}\text{GCF} = 3$ $\left.\begin{array}{l}m^3 = m\bullet m\bullet m\\ 2m^2 = 2\bullet m\bullet m\end{array}\right\}\text{GCF} = m\bullet m = m^2$ Then show how this applies to polynomials. $3m + 6 = 3(m + 2)$ $m + 2$ 3 

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Paper and Pencil**

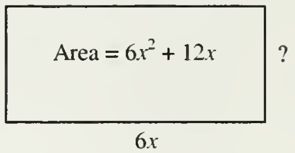
Paper and Pencil

1. Find the factors.
   a. $10x + 5$
   b. $2x^2 + 14x$
   c. $12x - 18y$
   d. $9c^3d + 5c^2 - 7c^2d$
   e. $5b^2c^2d^2 + 15bc - 10bcd^2$

2. The area of a rectangle is $10x^3y^2 + 5xy^2 - 15x^2y$ and the width is $5xy$. Find the length and the perimeter.

3. Use algebra tiles to factor $4x^2 - 8x$.

4. Find the missing expression for the width.



5. Complete the table.

Polynomial	GCF	Other Factor
$27r^3 - 18r^2 + 9r$	$9r$	
$14abc - 21ab^2$	$7ab$	
	$8y^2$	$3xz - 2y - 1x$
$2x^2y^2 + 8xy$		$xy + 4$
$6y^2 - 3xy + 9x^2y$		$2y - x + 3x^2$
	$4a$	$2a - b + 6$
$10x - 5y + 15$	5	

STRAND: PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

GENERAL OUTCOME

Generalize arithmetic operations from the set of rational numbers to the set of polynomials.

SPECIFIC OUTCOME

12. Factor trinomials of the form $ax^2 + bx + c$ where $a = 1$, or of the form $ax^2 + abx + ac$. [PS, R] (9–13)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Addison-Wesley Mathematics 10*, pp. 98–100
- *Interactions 9*, pp. 162–165
- *Mathpower 9*, pp. 195–197
- *Minds on Math 9*, pp. 358–365
- *TLE 9*, Factoring Polynomials, Student Refresher pp. 56–57, Teacher's Manual pp. 124–127

Previously Authorized Resources

- *Journeys in Math 9*, pp. 146–148
- *Math Matters: Book 2*, pp. 129–131, 134

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Since factoring is the reverse of multiplication, it may be helpful to review multiplication of simple binomials. Multiplication: $(x + 3)(x + 2) = x^2 + 5x + 6$ Factoring: $x^2 + 5x + 6 = (x + 3)(x + 2)$ The following steps could be used to factor a simple trinomial. 1. Split the x^2 term into its factors, and write them in the parentheses. $x^2 + 3x - 10 = (x \quad)(x \quad)$ 2. Determine all the factor pairs of the constant term. 10 and –1 –10 and 1 2 and –5 –2 and 5

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	3. Determine which pair has a sum equal to the coefficient of the middle term, and write them in the parentheses. $x^2 + 3x - 10 = (x - 2)(x + 5)$ 4. Check the answer by multiplying the factors. If the trinomial to be factored has a common factor in each of its terms, the common factor should be removed first. $2x^2 + 10x + 12 = 2(x^2 + 5x + 6)$ $= 2(x + 2)(x + 3)$

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

Paper and Pencil

1. Factor

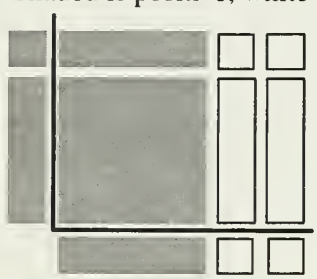
   a. $x^2 + 10x + 25$
   b. $x^2 - x - 12$
   c. $x^2 + x - 20$
   d. $x^2 + 8x + 16$
   e. $x^2 - 5x + 4$

2. Remove the GCF and factor fully.

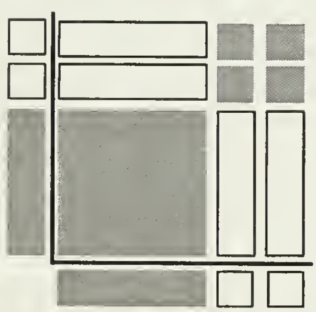
   a. $3x^2 + 6x - 9$
   b. $10x^2 - 60x + 80$
   c. $7x^2 + 42x + 56$
   d. $2x^2 - 4x - 30$
   e. $4x^2 - 8x - 60$

3. Identify the factors and the product for the diagrams below. (**Note**: Shaded is positive, white is negative)

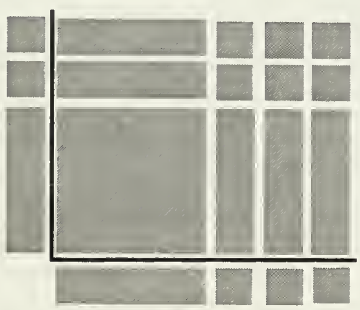
   a.



   b.



   c.



TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

4. State the dimensions and perimeter for each diagram.

   a. Area: $x^2 + 7x + 10$

   b. Area: $x^2 - 11x + 18$

   c. Area: $x^2 + 4x - 21$

5. Find all of the possible solutions for $\square$, assuming that the polynomial can be factored.

   a. $x^2 + \square x - 12$
   b. $x^2 - \square x + 14$
   c. $x^2 + \square x + 16$
   d. $x^2 - \square x - 20$

6. Complete the table.

Product	Factor	Factor
a. $x^2 - 3x - 18$	$(x + 3)$	
b.	$(x - 2)$	$(x + 5)$
c. $2x^2 + 22x + 20$	$2(x + 1)$	( )
d. $x^2 - 5x + 6$	$(x - 3)$	( )
e.	$3(x - 2)$	$(x + 8)$
f.	$(x + 1)$	$(x + 4)$
g.	$(2x + 3)$	$(2x + 3)$
h. $x^2 + 8x + 16$		

7. Change one term in the following trinomials to make the trinomial a perfect square.

   a. $x^2 - 25x + 100$
   b. $x^2 + 14x + 54$
   c. $x^2 - 16x - 64$
   d. $2x^2 + 18 + 81$

8. Identify the polynomials below that are perfect squares, and factor these polynomials.

   a. $x^2 - 22x + 121$
   b. $x^2 + 20x + 100$
   c. $x^2 + 2x + 4$

STRAND: PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

GENERAL OUTCOME	Generalize arithmetic operations from the set of rational numbers to the set of polynomials.
SPECIFIC OUTCOME	13. Factor polynomials of the form $A^2 - B^2$ where A and B are both monomial expressions. [PS, R] (9–11, 9–13)

MANIPULATIVES

- Algebra tiles

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Addison-Wesley Mathematics 10*, pp. 104–108
- *Interactions 9*, p. 163
- *Mathpower 9*, pp. 200–201
- *TLE 9*, Factoring Polynomials, Student Refresher pp. 56–57, Teacher's Manual pp. 124–127

Previously Authorized Resources
- *Math Matters: Book 2*, pp. 132–134

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	When faced with a factoring problem, it is important that students are able to recognize which type of factoring is involved—common, trinomial or difference of squares. When introducing difference of squares factoring, such vocabulary words as "square numbers" and "square roots" must be understood. Again, the concept of reverse operations can help. Multiplication: $(x+2)(x-2) = x^2 - 4$; $(x+5)(x-5) = x^2 - 25$ Factoring: $x^2 - 4 = (x+2)(x-2)$; $x^2 - 25 = (x+5)(x-5)$ The meaning of the phrase "difference of squares" should be discussed. It is also useful to discuss the reason that the middle term is missing in the product of $(x+5)(x-5) = x^2 - 25$. Once students understand these two concepts, most will be able to factor difference of squares binomials with little difficulty. Some may need the following steps.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	To factor a difference of squares: 1. Write the square root of the first term at the beginning of each parenthetical set. $x^2 - 16 = (x \quad)(x \quad)$ 2. Write the square root of the last term at the end of each parenthetical set. Include a plus sign in one and a minus sign in the other. $x^2 - 16 = (x + 4)(x - 4)$

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Paper and Pencil** 1. Factor a. $x^2 - 100$ b. $25x^2 - 49y^2$ c. $4a^2 - 9$ d. $16 - z^2$ 2. Remove the common factor, and factor fully. a. $81a^2b - 4b$ b. $32x^2 - 18y^2$ c. $27 - 12f^2$ d. $10y^2 - 10x^2$ e. $50 - 8x^2$ f. $36g^2 - 64$ **Performance** 1. Use algebra tiles to multiply $(x + 3)(x - 3)$, $(x + 2)(x - 2)$ and $(x + 5)(x - 5)$. Simplify each product. Describe the similarities in the three products. **Journal/Interview** 1. Why do you think the product of $(x - 6)(x + 6)$ is called a difference of squares? 2. Which of the following products are a difference of squares? Justify your answer. a. $x^2 - 121$ b. $125 + 100x^2$ c. $25x^2 - 16y$ d. $8x^2 - 50$ 3. Explain how to determine if a binomial product is a difference of squares.

STRAND: PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

GENERAL OUTCOME

Generalize arithmetic operations from the set of rational numbers to the set of polynomials.

SPECIFIC OUTCOME

14. Find the quotient when a polynomial is divided by a monomial. [PS, R] (9–14)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Addison-Wesley Mathematics 10*, pp. 76–78
- *Interactions 9*, pp. 122–127
- *Mathpower 9*, pp. 164–165, 175, 178, 190–191
- *Minds on Math 9*, pp. 292–293, 334–337, 348–349
- *TLE 9*, Factoring Polynomials, Student Refresher pp. 56–57, Teacher's Manual pp. 124–127

Previously Authorized Resources

- *Journeys in Math 9*, pp. 138–141
- *Math Matters: Book 2*, pp. 114–117, 120

TECHNOLOGY CONNECTIONS

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

Students should be exposed to the evaluation of polynomials (Patterns and Relations Specific Outcome 6) before and after they have been simplified so that the advantage of simplifying prior to evaluating can be emphasized. It is through such comparisons that students can see a need for simplifying.

There are two methods for division of a polynomial by a monomial. One method is to break the polynomial apart and solve individual monomial division problems; e.g., $\frac{3x+12}{3}=\frac{3x}{3}+\frac{12}{3}=x+4$.[1]

The second method is to factor the polynomial and simplify.

$$\frac{3x+12}{3}=\frac{\cancel{3}(x+4)}{\cancel{3}}=x+4$$

1. Divide $(3m^3+8m^2-5m)$ by $4m$.

   Solution—Method 1

$$\begin{aligned}\frac{3m^3+8m^2-5m}{4m}&=\frac{3m^3}{4m}+\frac{8m^2}{4m}+\frac{-5m}{4m}\\&=\frac{3m^2}{4}+2m+\frac{-5}{4}\\&=\frac{3}{4}m^2+2m+\frac{-5}{4}\\&=\frac{3}{4}m^2+2m-\frac{5}{4}\end{aligned}$$

[1] Adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 9 Implementation Draft.*

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Solution—Method 2

$$\frac{3m^3+8m^2-5m}{4m}$$

$$=\frac{\cancel{m}(3m^2+8m-5)}{4\cancel{m}}$$

$$=\frac{3m^2+8m-5}{4} \quad \text{or} \quad \frac{3}{4}m^2+2m-\frac{5}{4}$$

Verification

Verify by substituting $m = 10$.

LS	RS
$\frac{3(10)^3+8(10)^2-5(10)}{4(10)}$	$\frac{3}{4}(10)^2+2(10)-\frac{5}{4}$
$=\frac{3000+800-50}{40}$	$=\frac{3}{4}(100)+2(10)-\frac{5}{4}$
$=\frac{3750}{40}$	$=75+20-\frac{5}{4}$
$=\frac{375}{4}$ or $93\frac{3}{4}$	$=93\frac{3}{4}$

LS = RS

2. Divide $(8x^3 + 4x^2 - 4x)$ by $-4x$.

Solution—Method 1

$$\frac{8x^3+4x^2-4x}{-4x}=\frac{8x^3}{-4x}+\frac{4x^2}{-4x}+\frac{-4x}{-4x}$$
$$=-2x^2-x+1$$

Solution—Method 2

$$\frac{8x^3+4x^2-4x}{-4x}$$

$$=\frac{\cancel{4}x(2x^2+x-1)}{-\cancel{4}x}$$

$$=-2x^2-x+1$$

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Verification Verify by substituting $x = 10$.

LS	RS
$\dfrac{8(10)^3 + 4(10)^2 - 4(10)}{-4(10)}$	$-2(10)^2 - (10) + 1$
$= \dfrac{8000 + 400 - 40}{-40}$	$= -200 - 10 + 1$
$= \dfrac{8360}{-40}$	$= -209$
$= -209$	

LS = RS

3. Division is the same as multiplying by the reciprocal.

$$\frac{a+b}{c} = \frac{1}{c}(a+b)$$

By applying the distributive property, the problem is reduced to one of dividing monomials.

$$\frac{1}{c}(a+b) = \frac{1}{c} \bullet a + \frac{1}{c} \bullet b$$
$$= \frac{a}{c} + \frac{b}{c}$$

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Paper and Pencil**

1. a. Find the quotient of $\dfrac{12x^3 - 16x^2 + 8x}{4x}$ by dividing each term in the numerator by the term in the denominator.
   b. Find the quotient by factoring the numerator first. What do you notice?

2. Factor the numerator, then divide.
   a. $\dfrac{6x^3 + 4x^2 + 2x}{2x}$
   b. $\dfrac{4y^5 + 8y^3 - 2y^2}{2y^2}$

3. Perform the following divisions. Verify your answers by substituting $x = 10$.
   a. $(4x^3 + 2x^2 - 6x) \div 2x$
   b. $(6x - 12) \div (-3)$
   c. $\dfrac{4x^2 + 6x - 8}{5}$
   d. $\dfrac{8x^3 + 12x^2 - 4x}{-4x}$

4. Perform the following divisions.
   a. $(6x^3 + 4x^2 + 8x) \div 3x$
   b. $(12x^4 + 8x^3 - 16x^2 + 4x) \div (-4x)$
   c. $(x^3 - 9x) \div x$
   d. $\dfrac{-12y^3 + 9y^2 - y}{-3y}$
   e. $\dfrac{24 - 8m + 4m^2}{4}$
   f. $\dfrac{14a^2b^3 + 7a^2b^2 - 7ab^2}{7ab}$

5. a. Evaluate the expression $(4y^3 - 12y^2 + 8y) \div 4y$ for $y = -3$.
   b. Perform the division first, and then evaluate the expression once again for $y = -3$.
   c. Compare the two results.
   d. Was it simpler to evaluate the expression before or after division? Explain.

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	

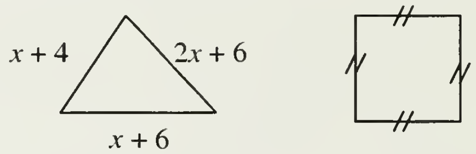
6. Simplify the expression:

$$\frac{2x^2 + 6x}{2x}$$

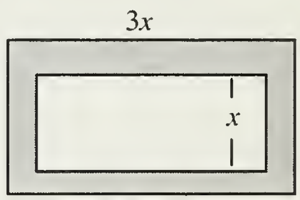
   a. Find the value of the expression for $x = 6$, by replacing x in the original expression.
   b. Find the value of the expression for $x = 6$, by replacing x in the simplified expression.
   c. Compare the two results. What do you notice?

Reproduced with permission from *Atlantic Canada Mathematics Curriculum: Grade 9 Implementation Draft.*

7. The perimeter of the triangle is equal to the perimeter of the square. Find the length of one side of the square in terms of x.



8. The inside rectangle in the diagram below is a flower garden. The shaded area is a concrete walk-way around it. The area of the flower garden is given by the expression $2x^2 - 4x$, and the area of the walk-way is $x^2 + 19x$.



   a. What is the total area of the flower garden and the concrete walk-way?
   b. Use the information provided to find an expression for each of the missing dimensions of each rectangle.
   c. If $x = 2.3$ m, find the dimensions and area of the flower garden.

Adapted with permission from *Atlantic Canada Mathematics Curriculum: Grade 9 Implementation Draft.*

STRAND: PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

GENERAL OUTCOME

Solve and verify linear equations and inequalities using one variable.

SPECIFIC OUTCOME

15. Illustrate the solution process for a one-step, single variable, first-degree equation, using concrete materials or diagrams.
    - $x + a = b$
    - $x - a = b$
    - $ax = b$
    - $\frac{x}{a} = b$

    [CN, PS, V] (7–7)

MANIPULATIVES

- Algebra tiles

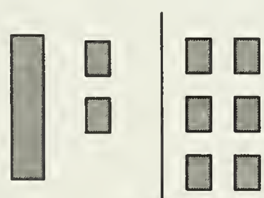
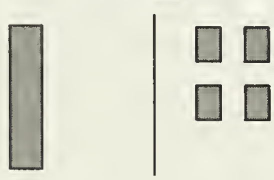
SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Interactions 7*, pp. 223–227
- *Interactions 8*, pp. 296–297
- *Mathpower 7*, pp. 228–235
- *Mathpower 8*, pp. 172, 180–183
- *Mathpower 9*, pp. 72–83
- *Minds on Math 7*, pp. 331–334, 338–341
- *Minds on Math 8*, pp. 371–376
- *Minds on Math 9*, pp. 133–138
- *TLE 8*, Linear Equations (1 Step Solution), Student Refresher pp. 44–45, Teacher's Manual pp. 100–103

Previously Authorized Resources
- *Journeys in Math 8*, pp. 354–357
- *Journeys in Math 9*, pp. 164–165

TECHNOLOGY CONNECTIONS

<table>
<tr><th></th><th>INSTRUCTIONAL STRATEGIES/SUGGESTIONS</th></tr>
<tr><td>Teaching Notes</td><td>Use algebra tiles to pictorially show one-step equations.

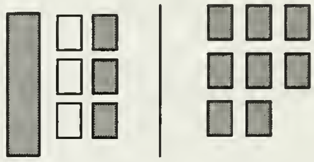
1. $x + 2 = 6$

(take away 2 from each side)

$x = 4$</td></tr>
</table>

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

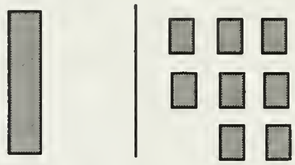
2. $x - 3 = 5$

(add 3 to each side)

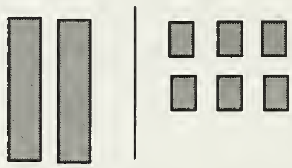


(combining, 3 sets of cancel out)

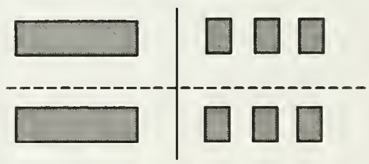
$x = 8$



3. $2x = 6$



(regroup for 1)



$x = 3$

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Journal/Interview** 1. Have students describe how they would set up specific equations with algebra tiles. 2. Have students describe the manipulation of tiles to solve these equations. **Paper and Pencil** 1. Use algebra tiles or diagrams to demonstrate the solution of the following. a. $a + 5 = 2$ b. $b - 3 = 6$ c. $c + 2 = -3$ d. $5d = -15$ e. $-3e = -12$

STRAND: PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

GENERAL OUTCOME

Solve and verify linear equations and inequalities using one variable.

SPECIFIC OUTCOMES

16. Solve and verify one-step linear equations of the form:
    - $x + a = b$
    - $\frac{x}{a} = b$
    - $ax = b$

    where a, b and c are integers, using a variety of techniques.
    [PS, R] (7–8)
18. Solve and verify one- and two-step first degree equations of the form:
    - $\frac{x}{a} + b = c$
    - $ax + b = c$

    where a, b and c are integers.
    [PS, V] (8–5)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Interactions 7*, pp. 219–231
- *Interactions 8*, pp. 291–305
- *Interactions 9*, pp. 226–231
- *Mathpower 7*, pp. 200–201, 228–235, 244
- *Mathpower 8*, pp. 176–183, 188–193, 196, 198
- *Mathpower 9*, pp. 70–83, 88–91
- *Minds on Math 7*, pp. 344–363
- *Minds on Math 8*, pp. 372–385
- *Minds on Math 9*, pp. 133–143
- *TLE 8*, Linear Equations (1 Step Solution), Student Refresher pp. 44–45, Teacher's Manual pp. 100–103
- *TLE 8*, Solving Problems, Student Refresher pp. 48–49, Teacher's Manual pp. 108–111

Previously Authorized Resources
- *Journeys in Math 8*, pp. 348–359
- *Journeys in Math 9*, pp. 162–167

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS[1]
Teaching Notes	Students need to understand three essential concepts when solving 2-step, single-variable, first-degree equations. • The objective is to finish with x equal to a value. • The equation must always balance. • To move a number, undo it by using the inverse operation; that is, add the opposite quantity to a term or multiply by the inverse of the coefficient.

[1] The Instructional Strategies/Suggestions for these specific outcomes are reproduced, by permission, from Manitoba Education and Training. *Grades 5 to 8 Mathematics: A Foundation for Implementation*. Winnipeg, MB: Manitoba Education and Training, 1997.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	When students are familiar with solving equations using concrete materials, ask them to solve equations using formal methods, such as the following:

1. $x + a = b$, where a is positive or negative

$$\begin{aligned} x - 3 &= 7 \\ x - 3 + 3 &= 7 + 3 \\ x + 0 &= 10 \\ x &= 10 \end{aligned} \qquad \text{OR} \qquad \begin{aligned} x - 3 &= 7 \\ \underline{\quad +3} &\underline{= +3} \\ x + 0 &= 10 \\ x &= 10 \end{aligned}$$

2. $ax = b$

$$\begin{aligned} 4x &= 20 \\ \frac{4x}{4} &= \frac{20}{4} \\ x &= \frac{20}{4} \\ x &= 5 \end{aligned}$$

3. $\frac{x}{a} = b$, where $a \neq 0$

$$\begin{aligned} \frac{x}{3} &= 4 \\ \frac{x}{3} \times 3 &= 4 \times 3 \\ x &= 4 \times 3 \\ x &= 12 \end{aligned}$$

4. $ax + b = c$

$$\begin{aligned} 4x + 7 &= 19 \\ \underline{\quad -7} &\underline{= -7} \\ 4x + 0 &= 12 \\ 4x &= 12 \\ \frac{4x}{4} &= \frac{12}{4} \\ x &= \frac{12}{4} \\ x &= 3 \end{aligned}$$

5. $\frac{x}{a} + b = c$, where $a \neq 0$

$$\begin{aligned} \frac{x}{3} + 5 &= 20 \\ \underline{\quad -5} &\underline{= -5} \\ \frac{x}{3} + 0 &= 15 \\ \frac{x}{3} \times 3 &= 15 \times 3 \\ x &= 15 \times 3 \\ x &= 45 \end{aligned}$$

Note: At first, ask students to use concrete materials to verify their solutions. Later, have them verify by substituting the value into the equation.

Verify $\frac{x}{3} + 5 = 20$, where $x = 45$

$$\begin{aligned} \frac{45}{3} + 5 &= 20 \\ 15 + 5 &= 20 \\ 20 &= 20 \end{aligned}$$

Left Side = Right Side

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Journal/Interview** 1. What does it mean to verify a solution? 2. Describe how to verify a solution. **Performance** 1. Provide students with a variety of 1- and 2-step equations to solve using concrete materials and have them record each step symbolically. Observations Check for the following: Does the student: • change the concrete form to its symbolic representation and vice versa? • solve 1-step, single-variable equations involving addition, subtraction, division or multiplication? • solve 2-step, single-variable equations involving: – addition and division? – subtraction and division? – addition and multiplication? – subtraction and multiplication? • solve the equation, given a value for the variable (solve for *y*, given *x*)? • determine the value of the variable, given the solution for the equation (solve for *x*, given *y*)? Observe and record student performance level: • independently with ease • independently with difficulty • only with assistance. Reproduced, by permission, from Manitoba Education and Training. *Grades 5 to 8 Mathematics: A Foundation for Implementation*. Winnipeg, MB: Manitoba Education and Training, 1997. **Paper and Pencil** 1. Solve and verify the following. a. $x + 9 = -5$ b. $x - 3 = -7$ c. $-2x = 18$ d. $\frac{x}{0.4} = -1.3$

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	2. Solve and verify the following. a. $2x + 6 = 22$ b. $7 - 5x = -27$ c. $\dfrac{x}{-2} - 4 = 9$ 3. Jacques found the solution for $2x + 7 = 13$ to be –3. Show how he could verify if his answer is correct.

STRAND: PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

GENERAL OUTCOME Solve and verify linear equations and inequalities using one variable.

SPECIFIC OUTCOME 17. Illustrate the solution process for a two-step, single variable, first-degree equation, using concrete materials or diagrams. [CN, PS, V] (8–4)

MANIPULATIVES

- Algebra tiles

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Interactions 8*, pp. 293–297
- *Interactions 9*, pp. 232–237
- *Mathpower 8*, pp. 188–189
- *Mathpower 9*, pp. 88–91
- *Minds on Math 8*, pp. 371–376
- *Minds on Math 9*, pp. 133–138
- *TLE 8*, Linear Equations (2 Step Solution), Student Refresher pp. 46–47, Teacher's Manual pp. 104–107

Previously Authorized Resources

- *Journeys in Math 8*, p. 358
- *Journeys in Math 9*, p. 166

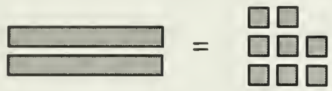
TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes Do not spend much time on this. It is very important for students to make the connection between what they do with the tiles and the symbolic algebra.	Using algebra tiles to demonstrate the solution of linear equations provides a visual aspect to an otherwise abstract activity. The analogy of keeping the teeter-totter balanced—doing the same thing to both sides of the equation—is very useful in promoting mathematically correct understanding of the equation-solving process. As students solve an equation, it is important that they know what their goal is: to isolate the variable. Keeping this in mind will help them know what needs to be eliminated. Students also need to understand the zero property. The following steps could be used to solve two-step linear equations. 1. Arrange the appropriate tiles to represent the equation. $2x - 3 = 5$ 

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	

2. Eliminate the three negative unit tiles from the left side by adding three positive unit tiles to each side.

3. Remove the zero pairs from the equation.



4. Arrange the unit tiles into two equal rows—because there are two x tiles on the left.

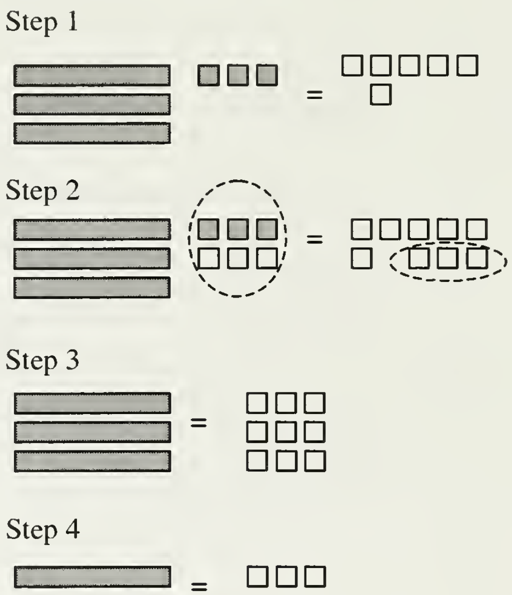
5. Remove one entire row from the equation. What remains is the answer ($x = 4$).

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

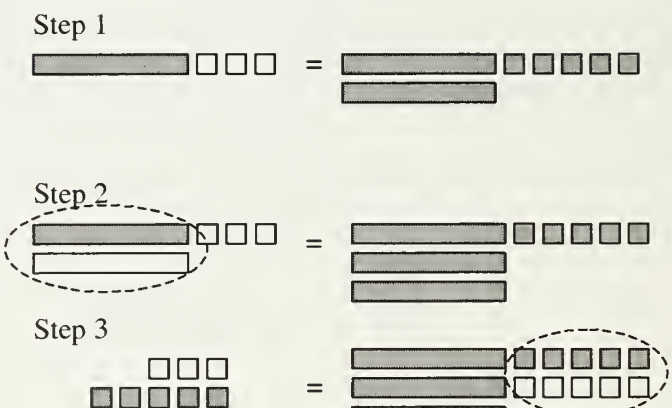
Paper and Pencil

1. Write the equivalent algebra equation for each diagram below. Describe each step in the solution of the equation.



2. Use algebra tiles to solve the equations.
   a. $3x - 5 = 10$
   b. $2x - 1 = 3x + 5$
   c. $6 = 2x - 3 + x$
   d. $5x - x + 1 = 3x - 2$
   e. $-2x + 5 = -x - 3$

3. Rob used tiles to solve the equation $x - 3 = 2x + 5$. Do an error analysis on the steps he used to solve for x.

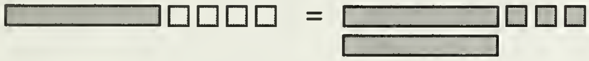


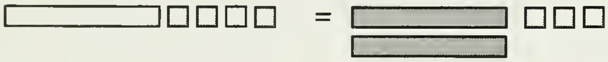
	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	

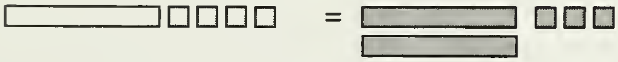
Step 4

Step 5

4. Which diagram satisfies the equation $-x - 4 = 2x + 3$?

a. 

b. 

c. 

d. 

STRAND: PATTERNS AND RELATIONS (VARIABLES AND EQUATIONS)

GENERAL OUTCOME Solve and verify linear equations and inequalities using one variable.

SPECIFIC OUTCOME

19. Solve and verify first-degree single-variable equations of the form:
    - $ax = b + cx$
    - $a(x + b) = c$
    - $ax + b = cx + d$
    - $a(bx + c) = d(ex + f)$
    - $\dfrac{a}{x} = b$

    where *a, b, c, d, e* and *f* are rational numbers (with a focus on integers), and use equations of this type to model and solve problem situations. [C, PS, V] (9–5)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Addison-Wesley Mathematics 10*, pp. 140–143, 158–161
- *Interactions 9*, pp. 232–241, 248, 250
- *Mathpower 8*, pp. 188–196
- *Mathpower 9*, pp. 88–108
- *Minds on Math 8*, p. 383
- *Minds on Math 9*, pp. 133–147, 150–159
- *TLE 9* Algebra Tiles Explorer
- *TLE 9*, Linear Equations 1–3, Student Refresher pp. 34–39, Teacher's Manual pp. 80–91

Previously Authorized Resources
- *Journeys in Math 9*, pp. 168–177, 186–189
- *Mathematics 9*, pp. 101–114

TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes All substitutions should be made into parentheses in order to evaluate correctly. It may be useful to explain this with the following example: $(-2)^2 = (-2) \times (-2)$ $= 4$ $-(2)^2 = -(2 \times 2)$ $= -4$ Avoid using -2^2 as students often misinterpret this.	1. $ax = b + cx$ $3x = 14 - 4x$ $3x + 4x = 14 - 4x + 4x$ $7x = 14$ $\frac{7x}{7} = \frac{14}{7}$ $x = 2$ Use inverse operations to collect like terms onto one side of the equation.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Verify by substitution: $3x = 14 - 4x$ $3(2) = 14 - 4(2)$ $6 = 14 - 8$ $6 = 6$ $\therefore x = 2$ is a solution 2. $a(x+b) = c$ $5(x+2) = 12$ $5x + 10 = 12$ $5x + 10 - 10 = 12 - 10$ $5x = 2$ $\frac{5x}{5} = \frac{2}{5}$ $x = \frac{2}{5}$ Use the distributive property to simplify the left-hand side. Verify by substitution: $5(x+2) = 12$ $5\left(\left(\frac{2}{5}\right) + 2\right) = 12$ $5\left(\frac{12}{5}\right) = 12$ $12 = 12$ $\therefore x = \frac{2}{5}$ is a solution 3. $ax + b = cx + d$ $x + 2 = 3x + 6$ $x - x + 2 = 3x - x + 6$ $2 = 2x + 6$ $2 - 6 = 2x + 6 - 6$ $-4 = 2x$ $\frac{-4}{2} = \frac{2x}{2}$ $-2 = x$ Use inverse operations to collect like terms. Verify by substitution: $x + 2 = 3x + 6$ $(-2) + 2 = 3(-2) + 6$ $0 = -6 + 6$ $0 = 0$ $\therefore x = -2$ is a solution

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	4. $a(bx + c) = d(ex + f)$

a. $$\begin{aligned} 3(-2x + 4) &= -4(x - 6) \\ -6x + 12 &= -4x + 24 \\ -6x + 6x + 12 &= -4x + 6x + 24 \\ 12 &= 2x + 24 \\ 12 - 24 &= 2x + 24 - 24 \\ -12 &= 2x \\ \frac{-12}{2} &= \frac{2x}{2} \\ -6 &= x \end{aligned}$$

Simplify first, by using the distributive property.

Use inverse operations to collect like terms.

Verify by substitution:

$$\begin{aligned} 3(-2x + 4) &= -4(x - 6) \\ 3[-2(-6) + 4] &= -4[(-6) - 6] \\ 3(12 + 4) &= -4(-12) \\ 3(16) &= 48 \\ 48 &= 48 \end{aligned}$$

$\therefore x = -6$ is a solution

b. $$\begin{aligned} \frac{3x - 6}{8} &= \frac{x + 2}{4} \\ 4(3x - 6) &= 8(x + 2) \\ 12x - 24 &= 8x + 16 \\ 12x - 8x - 24 &= 8x - 8x + 16 \\ 4x - 24 &= 16 \\ 4x - 24 + 24 &= 16 + 24 \\ 4x &= 40 \\ \frac{4x}{4} &= \frac{40}{4} \\ x &= 10 \end{aligned}$$

Find the cross products.

c. $$\begin{aligned} \frac{1}{3}(x + 5) &= \frac{2}{5}(x - 1) \\ 15\left(\frac{1}{3}\right)(x + 5) &= 15\left(\frac{2}{5}\right)(x - 1) \\ 5(x + 5) &= 3(2)(x - 1) \\ 5x + 25 &= 3(2)(x - 1) \\ 5x + 25 &= 6x - 6 \\ 5x - 5x + 25 &= 6x - 5x - 6 \\ 25 &= x - 6 \\ 25 + 6 &= x - 6 + 6 \\ 31 &= x \\ x &= 31 \end{aligned}$$

Multiply by the lowest common denominator.

Use the distributive property.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

In solving problems, students are asked to translate word problems into concrete models and then to symbolic equations. When students establish equations, they must:

- define the variables
- realize that word order is not necessarily the order found in the equation; e.g., the phrase "3 is subtracted from a number" implies $x - 3$
- know direct and indirect words for operations; e.g., add, sum, total.

Translating Word Problems into Symbolic Equations, and Solving the Equations

Provide students with problems such as the one below. As a class, discuss the various ways in which solutions to problems could be set up. For example, the variable could be any vehicle in question 1. Have each student create a similar problem, exchange his or her problem with a partner, solve, and then verify each other's solution.

1. There are 12 vehicles in a parking lot. There is 1 more van than trucks. There are 5 more cars than trucks. How many of each vehicle are in the parking lot?
   One Solution:

Vans	Trucks	Cars
n	$n - 1$	$(n - 1) + 5$

$$\begin{aligned} n + n - 1 + (n - 1) + 5 &= 12 \\ 3n + 3 &= 12 \\ -3 &= -3 \\ \hline 3n &= 9 \\ \frac{3n}{3} &= \frac{9}{3} \\ n &= 3 \end{aligned}$$

So 3 vans, 2 trucks and 7 cars make up the 12 vehicles in the parking lot. Check: $3 + 2 + 7 = 12$.

Note: Discuss other solutions where n = number of trucks and n = number of cars.

Reproduced, by permission, from Manitoba Education and Training. *Grades 5 to 8 Mathematics: A Foundation for Implementation*. Winnipeg, MB: Manitoba Education and Training, 1997.

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Paper and Pencil** 1. Solve and verify the following equations: a. $5x = 12 + 2x$ b. $7 - 2x - 3x - 1 = 21$ c. $2(x - 3) = x + 17$ d. $\frac{4}{x} = -2$ e. $0.3(x + 0.2) = 2(0.1x + 0.7)$ f. $\frac{4m}{6} - \frac{7}{2} = \frac{5m}{3}$ 2. The formula $G = 2.1n + 3.7$ can be used to find how long a traffic light stays green, where G is the green time in seconds and n is the number of vehicles that proceed per light cycle. a. How many vehicles proceed if green time is 40 seconds? b. If 50 cars can proceed, how long is the green light? 3. Reed has 21 nickels and dimes totalling $1.35. How many dimes does he have? 4. The sum of three consecutive even numbers is 96. Find the numbers.

STRAND: SHAPE AND SPACE (MEASUREMENT)

GENERAL OUTCOME

Solve problems, using perimeter, area, surface area and volume.

SPECIFIC OUTCOME

1. Estimate, measure and calculate the surface area and volume of any right prism, cylinder, cone or pyramid. [E, PS, T, R] (6–4, 7–2, 8–4)

Note: Specific outcome 1 is also addressed with specific outcome 10.

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Addison-Wesley Mathematics 10*, pp. 47–48, 76, 87, 106, 406–415
- *Interactions 8*, pp. 265–277
- *Interactions 9*, pp. 85, 88, 94, 157–158, 216–217, 286–287, 195–202
- *Mathpower 8*, pp. 248–257, 264–266
- *Mathpower 9*, pp. 266–278
- *Minds on Math 8*, pp. 420–449
- *Minds on Math 9*, pp. 444–469
- *TLE 9*, Volume and Surface Area, Student Refresher pp. 68–71, Teacher's Manual pp. 148–155

Previously Authorized Resources

- *Journeys in Math 8*, pp. 108–122
- *Journeys in Math 9*, pp. 208–215
- *Mathematics 9*, pp. 466–484

TECHNOLOGY CONNECTIONS

- Spreadsheet programs

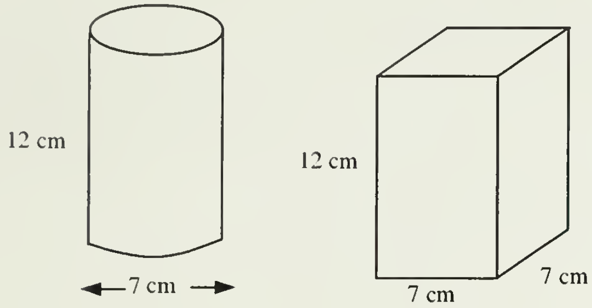
	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	In order for students to have a good understanding of surface area and volume, many will need to work with actual cylinders, cones, prisms and pyramids. Measuring these objects gives meaning to the formulas that they will use to calculate surface area and volume. Also, students must be able to identify the base(s) on these objects, since many of the formulas have base as one of the components. Following are some formulas that may be used to calculate surface area and volume. The calculation of surface area often requires a combination of two or more formulas. When using formulas, proper substitution techniques should be followed. Answers should include correct units of measurement.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	$A = lw$ (area of rectangle) $A = bh$ (area of parallelogram) $A = \frac{1}{2}bh$ (area of triangle) $A = \frac{h(a+b)}{2}$ (area of trapezoid) $A = \pi r^2$ (area of circle) $V = Bh$ (volume of prism or cylinder) Note: B = area of base $V = \frac{1}{3}Bh$ (volume of pyramid or cone) $A = 2\pi r^2 + 2\pi rh$ (surface area of cylinder) $A = \pi r^2 + \pi rs$ (surface area of cone) $c^2 = a^2 + b^2$ (Pythagorean theorem) Students may use 3.14 for π, or the π button on the calculator. The π button is the preferred method. **Sample Problem** Calculate the exposed surface area and the volume of a cone-shaped pile of sand whose radius is 14 m and whose vertical height is 10.5 m. **Solution** First calculate the slant height. $c^2 = a^2 + b^2$ $c^2 = 14^2 + 10.5^2$ $c^2 = 306.25$ $c = 17.5$ Then calculate the surface area. (Exclude the base.) $A = \pi rs$ $A = (3.14)(14)(17.5)$ $A = 769.3$ The exposed surface area is 769 m^2 or 770 m^2 if the π button is used. Then calculate the volume. $V = \frac{1}{3}Bh$ $V = \frac{1}{3}(\pi r^2)h$ $V = \frac{1}{3}(3.14 \times 14^2)(10.5)$ $V = 2154.04$ The volume is 2154 m^3 or 2155 m^3 if the π button is used.

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Paper and Pencil**

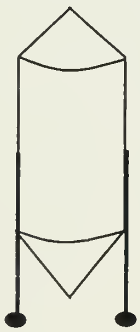
Paper and Pencil

1. A cardboard box is made so that a can fits snugly inside it. Both have the same height and width as shown in the diagram.

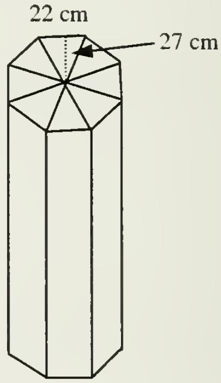
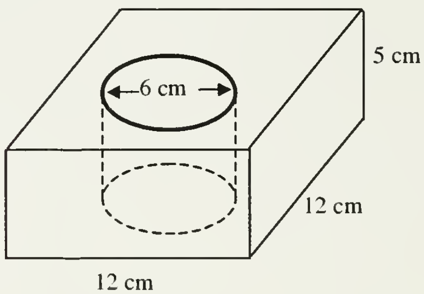


   a. Calculate the surface area of the can using the formula $A = 2\pi r^2 + 2\pi rh$.
   b. Using your answer from part a, estimate the surface area of the box.
   c. Calculate the surface area of the box, by adding the areas of each of its faces.
   d. Calculate the volume of the can, using the formula $V = \pi r^2 h$.
   e. Using your answer from part d, estimate the volume of the box.
   f. Calculate the volume of the box, using the formula $V = lwh$.

2. Many steel grain bins are made from a large cylinder, with both bases removed, and two cones, also with the bases removed. One cone is fastened at the bottom of the cylinder, and the other is fastened at the top as shown. The two cones are identical. The diameter of the cylinder is 3.2 m, and its height is 4.0 m. The cones each have a vertical height of 1.2 m and a slant height of 2.0 m.



   a. Calculate the total surface area of the grain bin.
   b. Calculate the total volume of the grain bin.

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	3. An ancient stone column has the shape of an octagonal prism. It was originally used to support a roof, but the roof has fallen, and the pillar now stands alone. The top of the pillar is a regular octagon, made up of eight identical isosceles triangles, each with a base of 22 cm and a height of 27 cm. The total height of the pillar is 6.8 m.  a. Calculate the total exposed surface area of the pillar. b. Calculate the volume of the pillar. Another pillar from the same building is round instead of octagonal. It has the same height, and its radius is 27 cm. It is also standing upright, with nothing covering the top. c. Using your answer from part a, estimate the exposed surface area of the round pillar. d. Calculate the exposed surface area of the round pillar. e. Using your answer from part b, estimate the volume of the round pillar. f. Calculate the volume of the round pillar. 4. A solid block of wood measuring 12 cm by 12 cm by 5 cm has a 6 cm diameter hole drilled through it as shown. The entire block is to be painted, including inside the hole.  a. Determine the surface area to be painted. b. Determine the volume of wood in the finished product.

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

It is assumed that the teacher and students are familiar with the operation of spreadsheets.

Computer Activity

1. A 355 mL aluminum pop can has a volume of 355 cm^3. Many different combinations of radius and height will result in this volume. Create a spreadsheet that will help determine which height and radius will require the least amount of aluminum (smallest surface area). Use the following column headings. Put formulas in columns C and D to calculate height and surface area from the given volume and radius.

	A	B	C	D
1	Volume	Radius	Height	Surface Area
2	355	2.0		
3	355	2.1		
4	355	2.2		
•	•	•		
•	•	•		
•	•	•		
25	355	4.3		

   a. Which height and radius combination results in the smallest surface area?
   b. How close is this to the actual size used by soft drink companies?
   c. Suggest some reasons for any differences between what the spreadsheet gives as the best combination and the measurements of an actual pop can.
   d. Modify the spreadsheet to determine the dimensions of a square-based, two-litre milk carton that will use the least amount of cardboard in its construction.

STRAND: SHAPE AND SPACE (MEASUREMENT)

GENERAL OUTCOME

Solve problems, using perimeter, area, surface area and volume.

SPECIFIC OUTCOME

2. Demonstrate concretely, pictorially and symbolically that many rectangles are possible for a given perimeter or a given area. [CN, R] (6–7)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Interactions 9*, pp. 203–205, 207
- *Mathpower 8*, pp. 213, 221
- *Minds on Math 8*, pp. 305–313
- *Minds on Math 9*, pp. 450–452
- *TLE 9*, Area and Perimeter, Student Refresher pp. 72–73, Teacher's Manual pp. 156–159

Previously Authorized Resources

- *Journeys in Math 8*, p. 93

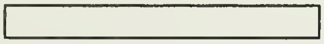
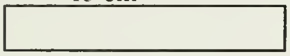
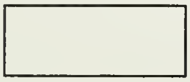
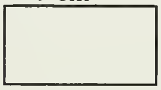
TECHNOLOGY CONNECTIONS

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

This outcome should not require much time. Depending on your students, you may be able to just refresh students' memories about these relationships.

1. Use a table/chart to show as many rectangles as possible with a perimeter of 30 cm and sides of whole numbers.

Diagram	w (cm)	l (cm)	$P = 2l + 2w$ (cm)
14 cm, 1 cm 	1	14	30
13 cm, 2 cm 	2	13	30
12 cm, 3 cm 	3	12	30
11 cm, 4 cm 	4	11	30
10 cm, 5 cm 	5	10	30
9 cm, 6 cm 	6	9	30
8 cm, 7 cm 	7	8	30

   a. Could there be other measurements?
   b. Could we continue on with widths greater than lengths? Would it make a difference?

2. Use a table/chart to show that many rectangles can have an area of 400 cm^2.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	3. Given that the perimeter of a rectangle is 30 cm, students could determine all possible whole number, or mixed number, dimensions of this rectangle and calculate the area for each set of dimensions. How does area vary when the perimeter is fixed? Describe the shape that gives the maximum area and the shape that gives the smallest area. 4. What happens to the area of a regular polygon as the number of sides increases? Given a perimeter of 36 cm, what is the area when the figure has 4 sides? 6 sides? 8 sides? 5. Determine the effect on the area of a polygon when its sides are doubled, tripled, quadrupled.... Can you make a rule for this? 6. Determine the effect on the area of a circle if its perimeter is doubled or halved. Can you make a rule for this?

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Journal/Interview** 1. Can many rectangles with different lengths and widths have the same perimeter and area? 2. Can the area of a circle or square vary if the circumference or perimeter of each is fixed? Explain. 3. Is there a relationship between areas of squares and circles that have the same perimeter? Elaborate. **Paper and Pencil** 1. Find as many whole number lengths and widths for the following rectangles: a. perimeter of 70 cm b. area of 500 cm^2 c. perimeter of 195 cm d. area of 144 m^2 2. Jordan ties his dog to the basketball pole in his yard while he is at school. The rope he uses is 9 m long. What is the maximum yard area in which the dog can roam?

STRAND: SHAPE AND SPACE (MEASUREMENT)

GENERAL OUTCOME

Solve problems, using right angle triangles.

SPECIFIC OUTCOME

3. Use the Pythagorean relationship to calculate the measure of the third side, of a right triangle, given the other two sides in 2-D applications. [PS] (8–2)

MANIPULATIVES

- Ruler
- Grid paper

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Addison-Wesley Mathematics 10*, pp. 25–29, 190, 388
- *Interactions 8*, pp. 246–249
- *Interactions 9*, pp. 196–200
- *Mathpower 8*, pp. 204–209
- *Mathpower 9*, p. 227
- *Minds on Math 8*, pp. 478–489
- *Minds on Math 9*, pp. 230, 310–311
- *TLE 8*, The Pythagorean Relationship, Student Refresher pp. 54–55, Teacher's Manual pp. 120–123
- *TLE 8*, Problem Solving: Using the Pythagorean Relationship, Student Refresher pp. 56–57, Teacher's Manual pp. 124–127

Previously Authorized Resources

- *Journeys in Math 8*, pp. 359–361
- *Journeys in Math 9*, pp. 111–114
- *Math Matters: Book 2*, pp. 54–59
- *Mathematics 9*, pp. 338–343, 468, 471, 473

TECHNOLOGY CONNECTIONS

- Scientific calculator

Strand: Shape and Space (Measurement)
Specific Outcome: 3. Use the Pythagorean relationship to calculate the measure of the third side, of a right triangle, given the other two sides in 2-D applications. [PS] (8–2)

Teaching Notes

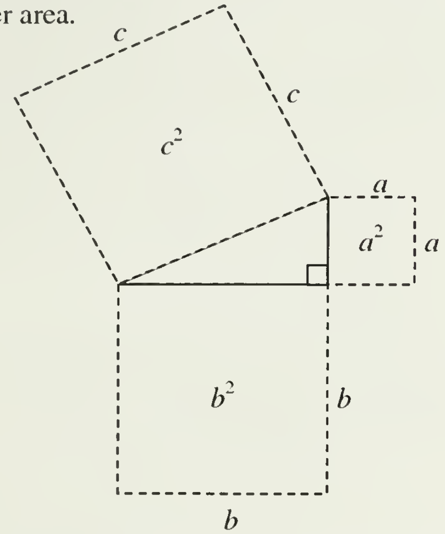
Students should also be encouraged to use rounding that is applicable to the question. For example, an answer of 152.425 km as a distance between two towns would more likely be expressed as 152 km.

Rounding by using significant digits may cause confusion, and although measurement is an important component of Applied Mathematics 10, the emphasis in this outcome should be on the use of Pythagorean theorem. Directions to students could include "round answer to the nearest tenth" or "express answer as a whole number."

Both imperial and SI units are used in Applied Mathematics 10. Examples should include both measurement systems.

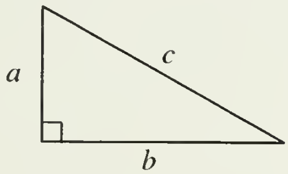
INSTRUCTIONAL STRATEGIES/SUGGESTIONS

1. Develop the Pythagorean relationship using grid paper.
   a. Draw a right triangle.
   b. Draw a square on each side of the triangle as shown.
   c. Compare/relate areas—the two smaller areas combined give the larger area.



Cut the squares out. Either align the larger square—on the hypotenuse—with the grid paper, or cut the smallest square into parts and fit these parts and the square on side b onto the square on the hypotenuse.

2. Pythagorean Theorem
   For any right triangle, $a^2 + b^2 = c^2$, where c is the hypotenuse.



$c = \sqrt{a^2 + b^2}$ Finding the hypotenuse when the other two sides that make the right angle are known.

$$\left.\begin{aligned} a &= \sqrt{c^2 - b^2} \\ b &= \sqrt{c^2 - a^2} \end{aligned}\right\}$$ Finding one of the shorter sides when the hypotenuse and the other side are known.

Suggestion: Students should practise rearranging the formula for the Pythagorean theorem to solve for a or b, rather than the teacher presenting individual formulas.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS

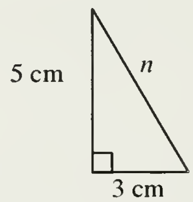
Teaching Notes

Pythagorean theorem is taught in Grade 8 and Grade 9. Less time should be spent on the actual theorem and more time on common errors, such as:

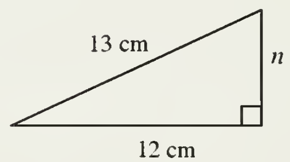
- not distinguishing the hypotenuse and always substituting the two known sides for a and b in the formula
- forgetting to find the square root—remind students that the formula solves for the square of the unknown side. Reinforcing the difference between square and square root is also important.

Students should be aware of these common triangles: 3, 4, 5; 5, 12, 13; 8, 15, 17 and any multiples of these (Pythagorean triples).

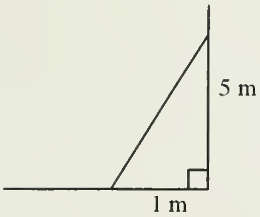
3. Examples: Find n. (Round to the nearest tenth.)



$$\begin{aligned} a^2 + b^2 &= n^2 \\ (3)^2 + (5)^2 &= n^2 \\ 9 + 25 &= n^2 \\ 34 &= n^2 \\ n &= \sqrt{34} \\ n &= 5.8 \text{ cm} \end{aligned}$$



$$\begin{aligned} a^2 + b^2 &= c^2 \\ n^2 + (12)^2 &= (13)^2 \\ n^2 + 144 &= 169 \\ n^2 + 144 - 144 &= 169 - 144 \\ n^2 &= 25 \\ n &= \sqrt{25} \\ n &= 5 \text{ cm} \end{aligned}$$

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	4. Solving Problems A ladder reaches 5 m up a wall. The bottom of the ladder is 1 m from the base of the wall. How long is the ladder? (Round to the nearest tenth.) Steps in Solution: 1. Draw diagram with information. 2. Substitute information into Pythagorean theorem. 3. Solve. 4. State answer.  $a^2 + b^2 = c^2$ $(1)^2 + (5)^2 = c^2$ $1 + 25 = c^2$ $c^2 = 26$ $c = \sqrt{26}$ $c = 5.099$ m The length of the ladder is 5.1 m, to the nearest 0.1 m.

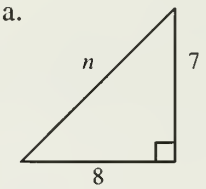
	TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

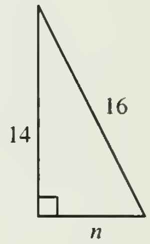
Paper and Pencil

1. Find n.

   a.



   b.



2. A 14 foot ladder is leaning against a tree. If the base of the ladder is three feet from the tree, how far up the tree will it reach?

3. A 40 m tower has a guy wire attached halfway up. Find the length of the guy wire if it is anchored 10 m from the base of the tower.

Journal/Interview

1. Describe the Pythagorean theorem in words.

2. Describe everyday situations that involve right angle triangles, and describe how you would find the measurements involved. Be specific; e.g., construction (mitred corner), surveying (distance that cannot be easily measured).

Portfolio

1. Collect pictures/articles from magazines and newspapers where right angle triangles appear. Measure the sides of the triangles, and show that the Pythagorean relationship is valid in every case.

STRAND: SHAPE AND SPACE (MEASUREMENT)

GENERAL OUTCOME

Solve problems, using right angle triangles.

SPECIFIC OUTCOME

4. Estimate, measure and classify angles as:
   - acute
   - obtuse
   - right
   - straight
   - reflex.

   [E] (6–10)

MANIPULATIVES

- Geometry set—ruler, protractor

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Interactions 7*, pp. 229–231
- *Mathpower 7*, pp. 264–267
- *Minds on Math 7*, pp. 368–377
- *Minds on Math 9*, pp. 183–193
- *TLE 7*, Classifying Angles, Student Refresher pp. 62–63, Teacher's Manual pp. 136–139

Previously Authorized Resources
- *Journeys in Math 8*, pp. 236–237
- *Journeys in Math 9*, pp. 229–231
- *Math Matters: Book 2*, pp. 290–294

TECHNOLOGY CONNECTIONS

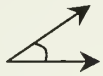
Teaching Notes

Students should have a strong intuitive understanding of the following angle measurements: 30°, 45°, 60°, 90°, 135° and 180°, and be able to draw a relatively accurate representation of each without a protractor. If this skill has been developed, then estimates of the measures of other angles will be more accurate.

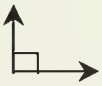
INSTRUCTIONAL STRATEGIES/SUGGESTIONS

1. Terminology:

   acute angle – angle that is less than 90°; e.g., 42°

   right angle – angle that is 90°

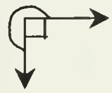
   obtuse angle – angle that is between 90° and 180°; e.g., 120°

   straight angle – angle that is 180°

   reflex angle – angle that is between 180° and 360°; e.g., 270°

2. Using a protractor and straightedge/ruler, construct a variety of angles; e.g., 47°, 116°, 90°, 285°, 180°.

3. Given a variety of angles measuring between 0° and 360°, use a protractor to find their measurement in degrees, and classify each.

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes Students may have difficulty finding reflex angles until they realize that the direction of rotation defines the type of angle. For example, they may perceive the corner of a door as 90°, but the angle around the corner of the door is 270°.	**Interview/Journal** 1. Describe an acute, right, obtuse, straight and reflex angle. 2. Look around the classroom and find an example of each angle. Which angle was easiest to find? 3. Explain how your perception of a three-dimensional object can change the classification of the angles you see. (See Teaching Notes.) **Paper and Pencil** 1. Without using a protractor, draw acute, right, obtuse, straight and reflex angles; and estimate their measurement. Use a protractor to check your results. 2. Construct the following angles, using a protractor, and classify them. a. 174° b. 13° c. 70° d. 90° e. 240° f. 168° g. 340°

STRAND: SHAPE AND SPACE (MEASUREMENT)

GENERAL OUTCOME Solve problems, using right angle triangles.

SPECIFIC OUTCOME 5. Explain the meaning of sine, cosine and tangent ratios in right triangles. [C, R] (9–1)

MANIPULATIVES
- Geometry set—ruler, protractor

SUGGESTED LEARNING RESOURCES

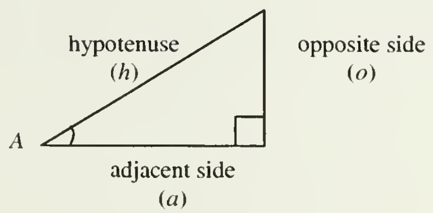
Currently Authorized Resources
- *Addison-Wesley Mathematics 10*, pp. 487–500
- *Interactions 9*, pp. 264–265, 268–269
- *Mathpower 9*, pp. 236–244
- *Minds on Math 9*, pp. 232–239, 246–252
- *TLE 9*, Ratios in Right Triangles, Student Refresher pp. 58–59, Teacher's Manual pp. 128–131

TECHNOLOGY CONNECTIONS
- Scientific calculator

<table>
<tr><th></th><th>INSTRUCTIONAL STRATEGIES/SUGGESTIONS</th></tr>
<tr><td>Teaching Notes</td><td>

1. Trigonometric ratios are the ratios of the lengths of sides of right angle triangles. Because similar triangles have the same angles, their shape remains the same and their sides have the same ratios. The same ratios of sides, therefore, will always give the same angle.

Terminology: For any given right angle triangle, the sides are labelled according to one of the given acute angles.



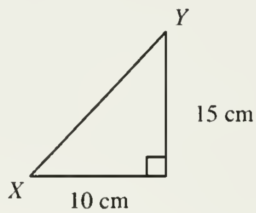
If the other acute angle is used, opposite and adjacent switch.

sin (sine) $A = \frac{o}{h}$

cos (cosine) $A = \frac{a}{h}$

tan (tangent) $A = \frac{o}{a}$

</td></tr>
</table>

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes Calculators should be set in degree mode. Graphing calculators, and many scientific calculators, use the order sin 30 to find sin 30. Other scientific calculators use 30 sin.	2. Use of Scientific Calculator with Trigonometry a. Given the angle → use "sin", "cos" or "tan" key and angle measurement ⇒ this gives the decimal equivalent of the ratio b. Given the sides → put sides in appropriate ratio and divide ⇒ this gives the decimal equivalent of the ratio, as above c. Given the decimal value→ use 2^{nd} function button, appropriate trigonometry button and decimal ⇒ this gives the angle 3. a. Draw accurate right angle triangles with an angle of 35° and various side lengths. Make a table of these lengths, put them into the three trigonometric ratios, and compare. b. For a right angle triangle with short sides of 10 cm and 15 cm: • Find all trigonometric ratios and put them in decimal form.  $\overline{XY} = \sqrt{15^2 + 10^2}$ $= 18$ $\sin X = \frac{15}{18} = 0.8\overline{3}$ $\cos X = \frac{10}{18} = 0.\overline{5}$ $\tan X = \frac{15}{10} = 1.5$ $\sin Y = \frac{10}{18} = 0.\overline{5}$ $\cos Y = \frac{15}{18} = 0.8\overline{3}$ $\tan Y = \frac{10}{15} = 0.\overline{6}$ • Find $\angle X$ and $\angle Y$. $\tan X = 1.5$ $\angle X = 56.3°$ $\sin Y = 0.\overline{5}$ $\angle Y = 33.7°$ **OR** $\angle Y = 90° - \angle X$ $= 90 - 56.3$ $= 33.7°$

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	See below

Journal/Interview

1. Describe the trigonometric ratios in words—what they are; i.e., ratios, and why they work as they do.

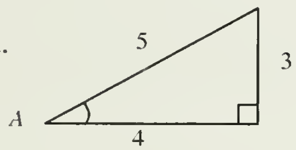
2. The unique characteristics of a right angle triangle make it useful in real-life circumstances. Support this statement.

3. Describe how you use a scientific calculator in trigonometry.

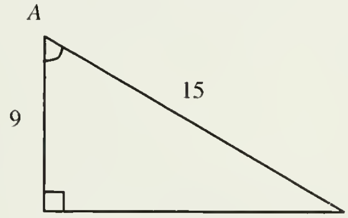
Paper and Pencil

1. Find the trigonometric ratios and their decimal equivalents for $\angle A$.

a.



b.



2. Use your calculator to determine the following:
   a. $\sin 37° =$
      $\tan 85° =$
      $\cos 16° =$
   b. $\sin A = 0.923$ $\quad \angle A =$
      $\cos B = 0.420$ $\quad \angle B =$
      $\tan C = 2.415$ $\quad \angle C =$

3. a. Draw accurate right angle triangles with an angle of 65° and various side lengths. Make a table of these lengths and their trigonometric ratios. Compare.
   b. Draw accurate right angle triangles with sides in appropriate ratios. Make a table of trigonometric ratios, and find the angles.

STRAND: SHAPE AND SPACE (MEASUREMENT)

GENERAL OUTCOME Solve problems, using right angle triangles.

SPECIFIC OUTCOME 6. Calculate an unknown side or an unknown angle in a right triangle, using trigonometric ratios. [PS, T, V] (9–3)

MANIPULATIVES

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Addison-Wesley Mathematics 10*, pp. 487–507
- *Interactions 9*, pp. 264–277
- *Mathpower 9*, pp. 236–247, 252–255
- *Minds on Math 9*, pp. 232–263, 266–269
- *TLE 8*, The Pythagorean Relationship, Student Refresher pp. 54–55, Teacher's Manual pp. 120–123
- *TLE 8*, Problem Solving: Using the Pythagorean Relationship, Student Refresher pp. 56–57, Teacher's Manual pp. 124–127
- *TLE 9*, Finding Unknown Sides and Angles, Student Refresher pp. 60–63, Teacher's Manual pp. 132–139

TECHNOLOGY CONNECTIONS

- Scientific calculator

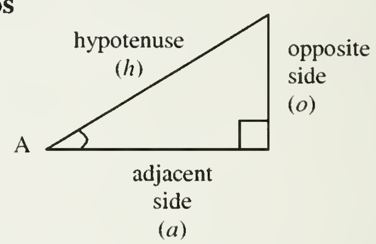
	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	

Review Trigonometric Ratios

$\sin \text{A} = \dfrac{o}{h}$

$\cos \text{A} = \dfrac{a}{h}$

$\tan \text{A} = \dfrac{o}{a}$



Emphasize the need to name sides correctly for either acute angle. Drawing each diagram and labelling it will provide a corresponding visual.

Process and Examples

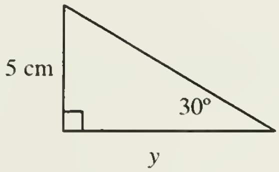
To solve right triangle problems:

a. Draw a diagram and label it.
b. Choose an appropriate trigonometric ratio—uses the information given.
c. Substitute information into the trigonometric ratio.
d. Calculate the answer.

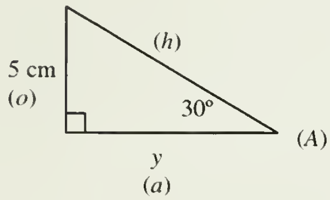
	INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

Example 1: Find y.



a.

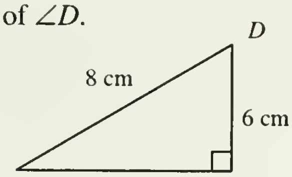


b. $\tan A = \dfrac{o}{a}$

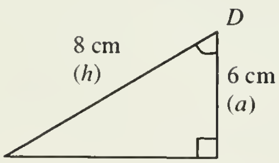
c. $\tan 30° = \dfrac{5}{y}$

d. $y = \dfrac{5}{\tan 30°}$
$= 8.660$
Side y is 8.660 cm

Example 2: Find the measure of $\angle D$.



a.

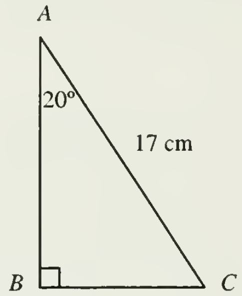


b. $\cos D = \dfrac{a}{h}$

c. $\cos D = \dfrac{6}{8}$

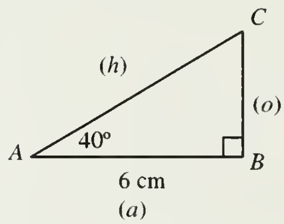
d. $\cos D = 0.75$ use 2^{nd} function
$\angle D = 41.4°$

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Example 3: Find the measure of $\angle C$.



Since $\angle B$ is 90°, $\angle A + \angle C = 90°$
$\therefore \angle C = 90° - 20° = 70°$

Example 4: A right angle triangle ABC has $\angle B = 90°$, $\angle A = 40°$ and $\overline{AB} = 6$ cm. Find the measure of $\overline{AC}$, $\overline{BC}$ and $\angle C$. Round answers to the nearest hundredth.



$$\cos A = \frac{a}{h}$$
$$\cos 40° = \frac{6}{h}$$
$$h = \frac{6}{\cos 40°}$$
$$= 7.8324$$
$\overline{AC}$ is 7.83 cm.

$$\tan A = \frac{o}{a}$$
$$\tan 40° = \frac{o}{6}$$
$$o = \tan 40° \times 6$$
$$= 5.0346$$
$\overline{BC}$ is 5.03 cm.

$\angle C = 90° - \angle A$
$\angle C = 90° - 40°$
$= 50°$
$\angle C$ is 50°.

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT

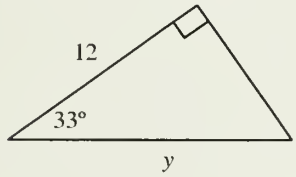
Teaching Notes

Journal/Interview

1. What is meant by opposite and adjacent sides? How are they related to the acute angles? Why is cos 60º equal to sin 30º?

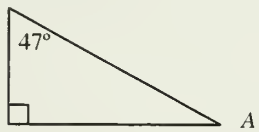
Paper and Pencil

1. Calculate y to three decimal places.

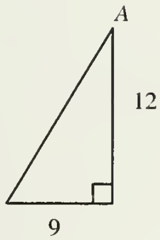


2. Find $\angle A$.

a.



b.



3. Triangle ABC has $\angle B = 90°$, $\overline{AB} = 16$ cm, and $\overline{BC} = 11$ cm. Find the measure of the missing angles and side. Round answers to the nearest hundredth of a centimetre, or to the nearest tenth of a degree.

STRAND: SHAPE AND SPACE (MEASUREMENT)

GENERAL OUTCOME Solve problems, using right angle triangles.

SPECIFIC OUTCOME

7. Model and then solve given problem situations involving only one right triangle. [PS, T, V] (9–4)

MANIPULATIVES

- Clinometer

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

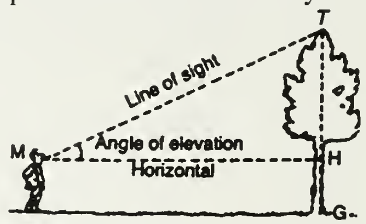
- *Addison-Wesley Mathematics 10*, pp. 492, 499, 500, 503–507
- *Interactions 9*, pp. 264–275
- *Mathpower 9*, pp. 238, 239, 241–247
- *Minds on Math 9*, pp. 236–239, 243–244, 249, 251, 252, 256–263
- *TLE 8*, Problem Solving: Using the Pythagorean Relationship, Student Refresher pp. 56–57, Teacher's Manual pp. 124–127
- *TLE 9*, Solving Right Triangles, Student Refresher pp. 64–67, Teacher's Manual pp. 140–147

TECHNOLOGY CONNECTIONS

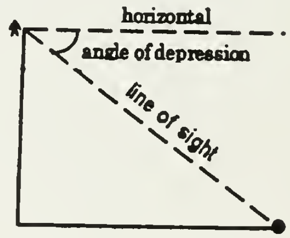
	INSTRUCTIONAL STRATEGIES/SUGGESTIONS[1]

Teaching Notes

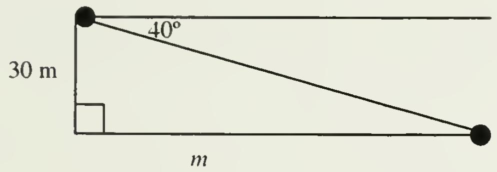
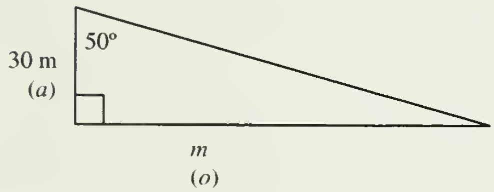
Angle of elevation—If you stand and look directly at the top of a tall tree, your line of sight slopes upwards from the horizontal. The angle between your line of sight and the horizontal is called the *angle of elevation* to the top of the tree from where you are standing.



Angle of depression—If you stand on a cliff and look down toward an object, the angle between the horizontal and the line of sight is called the *angle of depression.*



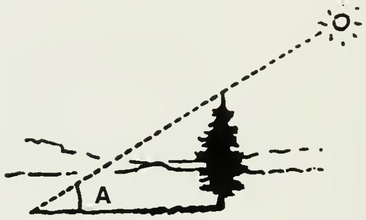
[1] The introductory Instructional Strategies/Suggestions information and Sample Questions 2 to 4 are reproduced, by permission, from Manitoba Education and Training. *Senior 1 Mathematics: A Foundation for Implementation.* Winnipeg, MB: Manitoba Education and Training, 1997.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Surveyors measure angles of elevation with an instrument called a *theodolite*. On a more basic level, a *clinometer* can be used by students to find the height of tall structures. Clinometers can be purchased or can be constructed from a protractor, straw, string and a plumb line. Sample Questions 1. Bill stood on a cliff overlooking a lake and saw a sailboat on the water. The cliff was 30 m high, and the angle of depression was 40°. How far out in the lake was the boat?   $\tan A = \frac{o}{a}$ $\tan 50° = \frac{m}{30}$ $\frac{\tan 50°}{1} = \frac{m}{30}$ $m = \frac{\tan 50° \times 30}{1}$ $m = 35.7526$ The boat was 35.75 m out in the lake.

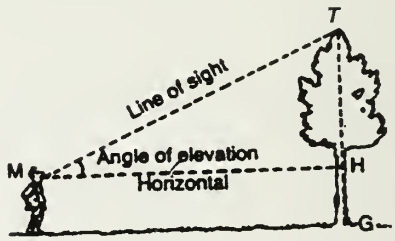
INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

2. A tree 2.50 m tall casts a shadow 4.36 m long. Calculate the angle of elevation of the sun to the nearest degree. (The angle is marked *A* in the diagram.)



3. Suppose that the distance *MH* in the picture is 10 m, and the angle of elevation is 25°.
   a. Calculate the distance *TH* , to the nearest 0.1 m.
   b. To find the height of the tree, you have to add the distance *HG*. This is the same as the height of the man's eyes above the ground. If his eyes are 1.6 m above the ground, how high is the tree?



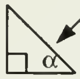
   c. The same man stands 25 m away from a flagpole and finds that the angle of elevation of the top of the pole is 41°. How tall is the pole?

4. Find the height of a tall structure close to the school using a clinometer and a measuring tape.

   a. Make a scale drawing to find the height.
   b. Use trigonometric ratios to find the height.

   Compare your answers and write a detailed report on your procedures and findings.

   There is another way to complete this task. What is it? Explain.

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Projects** 1. Have students measure various heights of objects without the use of a ladder; e.g., telephone poles, trees, flagpoles.  **Portfolios/Journals/Interviews** 1. Find examples of real-life situations that can be solved using trigonometry. 2. Find occupations that use this type of problem solving. **Paper and Pencil** 1. A staircase goes to the second floor of a house that is 12 m above the ground floor. The handrail is 16 m long. What is the angle of elevation of the handrail and the horizontal distance covered? 2. A tree casts a shadow of 14.3 m. At this point the angle of elevation to the top of the tree is 27°. How tall is the tree?

STRAND: SHAPE AND SPACE (3-D OBJECTS AND 2-D SHAPES)

GENERAL OUTCOME

Specify conditions under which triangles may be similar and use these conditions to solve problems.

SPECIFIC OUTCOME

8. Recognize when, and explain why, two triangles are similar, and use the properties of similar triangles to solve problems. [C, PS, R, T] (9–8)

MANIPULATIVES

- Ruler

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Addison-Wesley Mathematics 10*, pp. 480–486
- *Interactions 9*, pp. 258–262
- *Mathpower 9*, pp. 226–231
- *Mathpower 10*, pp. 226–235
- *Minds on Math 9*, pp. 208–223
- *TLE 9*, Similarity and Similar Triangles, Student Refresher pp. 74–77, Teacher's Manual pp. 160–167

Previously Authorized Resources
- *Math Matters: Book 2*, pp. 322–325
- *Mathematics 9*, pp. 390–396

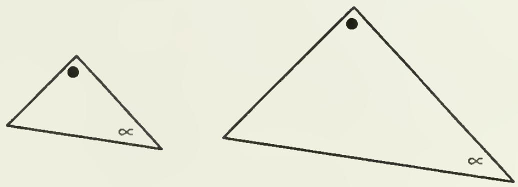
TECHNOLOGY CONNECTIONS

<table>
<tr><th></th><th>INSTRUCTIONAL STRATEGIES/SUGGESTIONS</th></tr>
<tr><td>Teaching Notes</td><td>

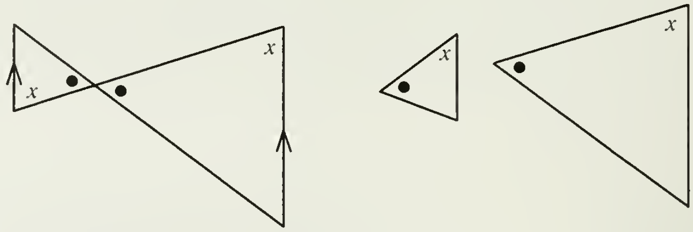
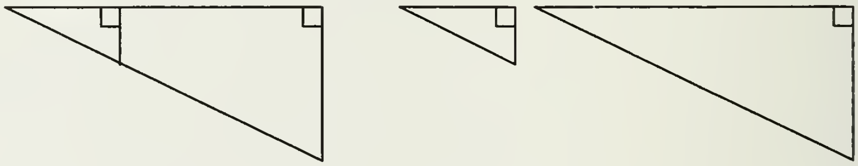
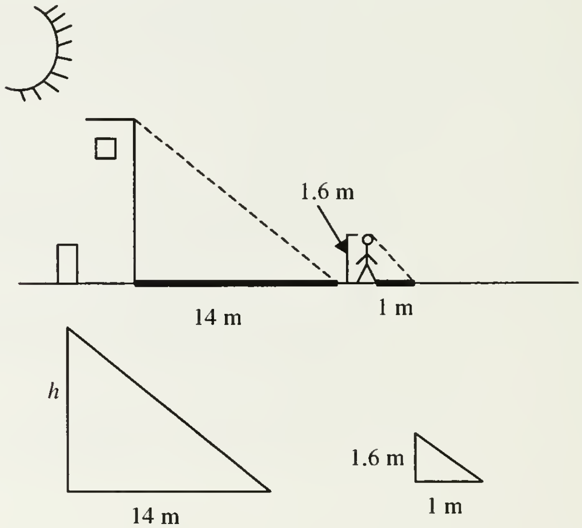
Investigation

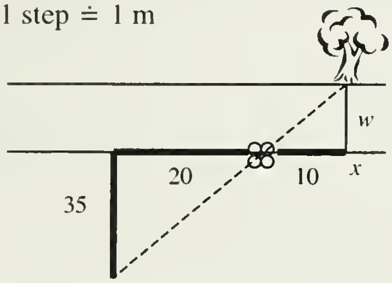
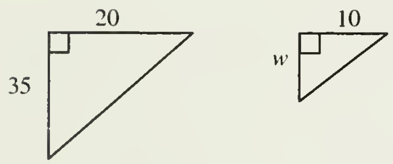
1. Terminology: Similarity means same shape but not necessarily the same size.
Similar triangles have corresponding angles that are congruent; i.e., makes the same shape, and corresponding sides that have the same ratio; i.e., proportional.
If the corresponding angles of triangles are congruent, this proves similarity and makes the corresponding sides proportional.

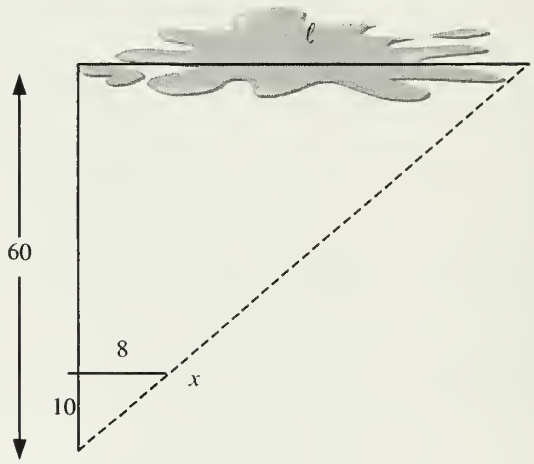
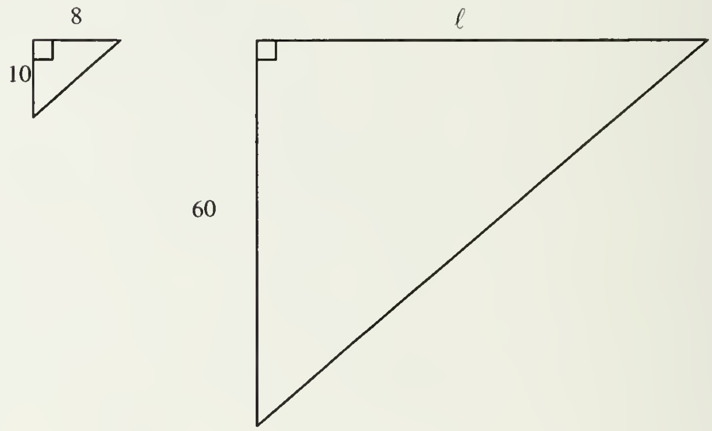
2. Examples of similar triangles.
a. Two separate triangles of the same shape.



</td></tr>
</table>

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	b. Bow tie.  c. Triangle within a triangle.  3. Real-life problem solving where similar triangles can be used. a. A building casts a shadow of 14 m. Tom is 1.6 m tall and casts a shadow of 1 m when standing next to the building. How tall is the building?  $\frac{h}{14}=\frac{1.6}{1}$ $h=\frac{14\times1.6}{1}$ $h=22.4$ The building is 22.4 m tall.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes An average step is 3 feet long for a 6 foot tall person. To do parts b and c of question 3, the person should take long steps.	b. Find the width of a river. 1. Sight a tree on the opposite bank. 2. Turn 90°, walk along the river for 10 steps, and make a pile of stones. 3. Continue from here for 20 steps. 4. Make a 90° turn from the river and walk until the pile of stones and the tree line up, counting steps (say 35). 5. You now have bow tie similarity. 1 step ≐ 1 m   $\frac{20}{35}=\frac{10}{w}$ $w=\frac{35\times 10}{20}$ $w=17.5$ The river is 17.5 m wide.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	c. Find the length of a pond. 1. Sight a post on the opposite side. 2. Turn at a right angle, and walk and count very long steps until you see the post with an unobstructed view. 3. Pound a stake into the ground at this point. 4. Walk back along your path for 10 very long steps. 5. Make a 90° turn right, and walk and count very long steps until the post and stake line up. 6. You now have a triangle within a triangle. 1 step $\doteq$ 1 m   $\frac{8}{10} = \frac{\ell}{60}$ $\ell = \frac{8 \times 60}{10}$ $\ell = 48$ The pond is 48 m long.

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	

Journal/Interview

1. How would you determine if two triangles are similar?

Portfolios

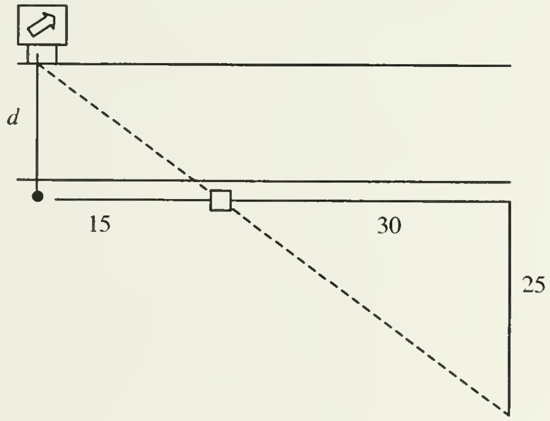
1. Go outside and measure a number of heights, using the shadow method; e.g., height of the school, flagpole, backstop, power pole.

Paper and Pencil

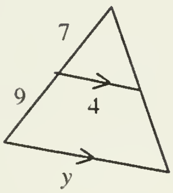
1. Answer true or false for similar triangles. Explain.
   a. They have the same shape.
   b. They have the same size.
   c. They have the same orientation.
   d. Corresponding sides are congruent.
   e. Corresponding angles are congruent.

2. A flagpole casts a shadow of 22.3 m. Bill is 1.75 m tall and casts a shadow of 3.1 m when standing next to the flagpole. How high is the flagpole?

3. Lee wants to find the distance across a highway. He sights a sign on the opposite side and then paces off distances as shown. How far is it across the highway?



4. Solve for y.



STRAND: SHAPE AND SPACE (TRANSFORMATIONS)

GENERAL OUTCOME

Create and analyze patterns and design, using symmetry, translation, rotation and reflection.

SPECIFIC OUTCOME

9. Draw designs, using ordered pairs, in all four quadrants of the coordinate grid. [PS, V] (7–13)

MANIPULATIVES

- Grid paper
- Ruler

SUGGESTED LEARNING RESOURCES

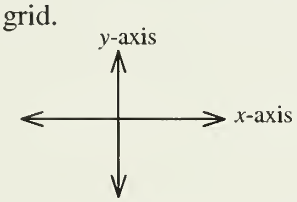
Currently Authorized Resources
- *Interactions 7*, pp. 91–93
- *Mathpower 7*, pp. 206–211
- *Mathpower 8*, pp. 159–161
- *Minds on Math 7*, pp. 268–269
- *TLE 7*, The Coordinate Plane, Student Refresher pp. 72–73, Teacher's Manual pp. 156–159
- *TLE 9*, Transformations Explorer
- *TLE 9*, Transformations on Grids, Student Refresher pp. 86–93, Teacher's Manual pp. 184–199

Previously Authorized Resources
- *Journeys in Math 8*, pp. 278–279
- *Journeys in Math 9*, pp. 308–309
- *Math Matters: Book 2*, pp. 140–142

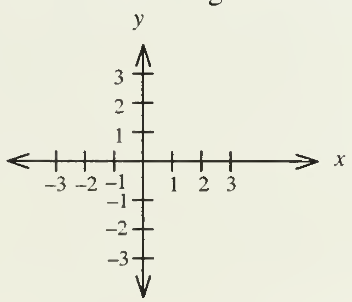
TECHNOLOGY CONNECTIONS

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	1. Develop the terminology and vocabulary using an overhead or chalkboard grid.

a.



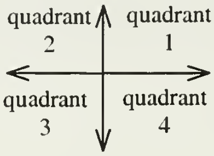
b. Present the idea of axes being number lines with a scale of integers.



INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

c. Axes divide the grid into four parts, called quadrants.



d. Where the axes join is called the origin, and it is given the ordered pair (0, 0).

e. Ordered pairs are used to locate specific points on the coordinate grid. The first number indicates a movement from the origin left (–) or right (+) and the second number indicates a movement from that point up (+) or down (–).

2. Examples Using a Grid
   a. Locate the following points.
      - A (3, 5)
      - B (–2, 7)
      - C (4, –1)
      - D (0, –3)
      - E $(-1, -5\frac{1}{2})$
      - F (–7, 0)

      Indicate the quadrant in which they lie.

   b. Pick some points and have students identify their ordered pairs.

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Portfolio** 1. Have students create their own picture on a grid and then develop the set of points that would produce their graphing picture. 2. Have students share their portfolio creations, and have others draw the pictures. **Journal/Interview** 1. Describe how to plot a particular ordered pair. 2. How can you always tell if an ordered pair is on: a. the x-axis? b. the y-axis? 3. Identify a way that you can determine the quadrant in which an ordered pair lies.

STRAND: SHAPE AND SPACE (TRANSFORMATIONS)

GENERAL OUTCOME

Create and analyze patterns and design, using symmetry, translation, rotation and reflection.

SPECIFIC OUTCOME

10. Draw and interpret scale diagrams:
    - enlargements
    - reductions.

    [PS, T] (8–11)

MANIPULATIVES

- Ruler
- Grid paper
- Pencil crayons/felt markers

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Addison-Wesley Mathematics 10*, pp. 321–327
- *Interactions 8*, pp. 65–72, 77
- *Interactions 9*, pp. 69–71, 73
- *Mathpower 8*, pp. 96–101
- *Mathpower 9*, pp. 309–311
- *Minds on Math 8*, pp. 134–152
- *Minds on Math 9*, pp. 406–411
- *TLE 8*, Enlargement and Reduction, Student Refresher pp. 72–73, Teacher's Manual pp. 156–159
- *TLE 9*, Dilatations and Similarity, Student Refresher pp. 90–91, Teacher's Manual pp. 192–195

Previously Authorized Resources

- *Journeys in Math 8*, pp. 182–186, 398–399
- *Journeys in Math 9*, pp. 406–411
- *Math Matters: Book 2*, pp. 205–209, 240–241

TECHNOLOGY CONNECTIONS

- *Geometer's Sketchpad*

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Refer to the program of studies for an elaboration of dilatations.

Investigation Connecting Specific Outcomes 1 and 10

Problem: If the dimensions of a rectangular prism were tripled, would the volume and surface area also be tripled?

Materials:
- 3-D package; e.g., candy box, juice box
- ruler
- grid paper (3 cm × 3 cm)
- pencil crayons/felt markers

Procedure:

- Open up the package into its net.
- Measure all of its sides.
- Calculate its surface area and volume.
- Draw a 1 cm × 1 cm grid on the net.
- Enlarge this net by reproducing the net onto the 3 cm × 3 cm grid paper.
- Measure the sides of the enlarged net.
- Calculate the enlarged surface area and volume.
- Reproduce all the logos and lettering of the 1 cm × 1 cm net onto the 3 cm × 3 cm.
- Cut out the enlarged net.
- Assemble the package.

Analysis:

a. Compare your results with the class by compiling a table.

Volume	
Original Size	Enlargement

Surface Area	
Original Size	Enlargement

b. By what scale factor does volume increase, if the dimensions were tripled?

c. By what scale factor does surface area increase, if the dimensions were tripled?

Extension:

What is the relationship between the volume and surface area of an object's original and enlarged size?

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

As students are involved in the activities, observe and note if they:

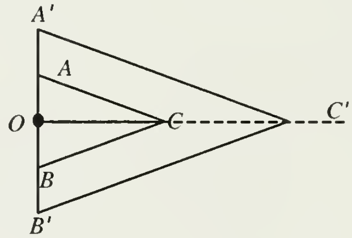
- know the geometric names of the solids, faces and geometric shapes
- measure dimensions accurately
- make reductions and enlargements accurately
- calculate surface area and volume from nets.

Sample Questions[1]

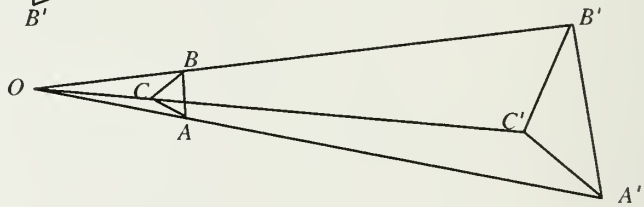
1. Give pairs of students a bucket of interlocking cubes.
   a. Show students a shape. Ask them to construct a shape which represents an enlargement by a factor of 3.
   b. Ask students to each make a shape, using exactly 6 cubes, exchange shapes with a partner, and ask their partner to create an enlargement of the shape by a factor of 2.

2. Determine the scale factor for each of the following:

   a.



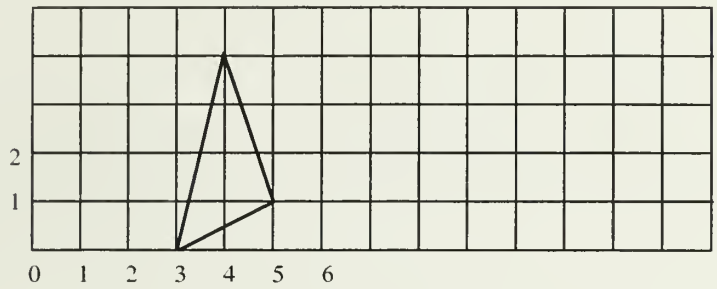
   b.

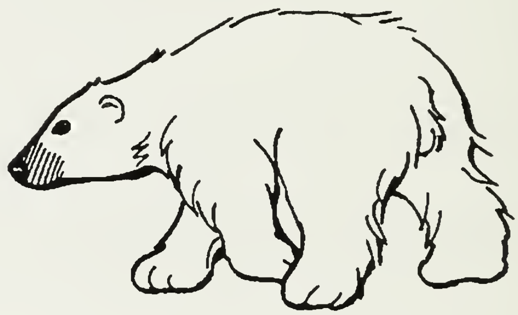


3. Reduce a figure by multiplying the x and y coordinates by the same fraction—between 0 and 1.
   a. Plot the figure and its image.
   b. Notice the differences in side lengths and areas.
   c. Notice that the shape and angles are the same.
   d. Identify the dilatation factor.

4. A photograph is 5 cm by 7 cm. It is enlarged by a scale factor of 2.5. What are the new dimensions?

[1] Sample Questions 1, 2, 4 and 5 are reproduced with permission from *Atlantic Canada Mathematics Curriculum: Grade 8*.

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	5. In the diagram below, using the origin as the centre of dilatation, a. enlarge the given figure by a scale factor of 2 b. reduce the given figure by a scale factor of $\frac{1}{2}$ c. record the vertices for each image 

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Performance** 1. Choose a cartoon to reduce in size. Draw a 3 cm × 3 cm grid onto the picture. Copy the contents of the cartoon onto 1 cm × 1 cm grid paper. **Project**[1] 1. The town council of Churchill, Manitoba wants to commission a giant statue of a walking polar bear based on the sketch shown. They want a statue that is 8 m high. Your company wants to bid on making the statue. The sketch is a reduction image of a real polar bear. Measure the height of the bear in the sketch, estimate the scale factor relative to the statue, then calculate the factor. Compare with a classmate and discuss whether as a company you would use the estimated or the calculated scale factor to produce the statue. Write a report detailing to the council the size of any 3 parts of the statue, such as diameter of the eye, width of head and length of body.  Extension: The density of the statue will be the same as that of the average polar bear. Investigate the average mass of polar bears. Using this mass, calculate the cost of the statue at $55 per kilogram and add it to your report. 2. Make a 2-legged animal out of interlocking cubes. Determine its surface area and volume. What is the ratio of surface area to volume? Now double the size of the animal in all 3 dimensions. What happens to the ratio of surface area to volume?

[1] Project questions 1 and 2 are reproduced, by permission, from Manitoba Education and Training. *Grades 5 to 8 Mathematics: A Foundation for Implementation.* Winnipeg, MB: Manitoba Education and Training, 1997.

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	**Paper and Pencil** 1. a. Plot *ABCD*, where $A = (2, 3)$, $B = (-5, 3)$, $C = (-5, 1)$ and $D = (2, -1)$. Using $(-2, 4)$ as the centre of dilatation, reduce this quadrilateral by a factor of $\frac{1}{2}$. b. Give the coordinates of the new quadrilateral $A'B'C'D'$. c. Is there a scale factor that would reduce or enlarge the quadrilateral so that it lies entirely in one quadrant? Explain your reasoning.

STRAND: STATISTICS AND PROBABILITY (DATA ANALYSIS)

GENERAL OUTCOME Develop and implement a plan for the display and analysis of data.

SPECIFIC OUTCOME 1. Read and interpret graphs that are provided. [C, E, PS, R] (6–7)

MANIPULATIVES
- Newspapers, magazines

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Addison-Wesley Mathematics 10*, pp. 245, 250–254
- *Interactions 7*, pp. 9–12, 20–22, 128, 205
- *Interactions 8*, pp. 11–13, 48, 160–171, 274, 312
- *Interactions 9*, pp. 95, 145
- *Mathpower 7*, pp. 326–341
- *Mathpower 8*, pp. 320–331
- *Mathpower 9*, pp. 130–131
- *Minds on Math 7*, pp. 202–215, 248
- *Minds on Math 8*, pp. 215, 220, 227–230, 296–297
- *Minds on Math 9*, pp. 56–63
- *TLE 7*, Reading and Interpreting Graphs, Student Refresher pp. 82–83
- *TLE 9*, Drawing Conclusions, Student Refresher pp. 102–103, Teacher's Manual pp. 216–219

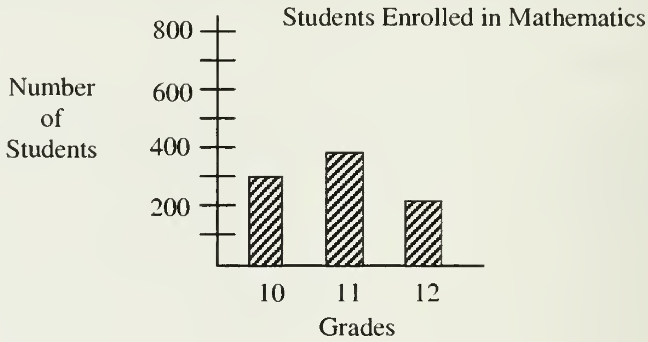
Previously Authorized Resources
- *Journeys in Math 8*, pp. 307, 310–318
- *Journeys in Math 9*, pp. 335–337, 342–348
- *Mathematics 9*, pp. 436–439

TECHNOLOGY CONNECTIONS

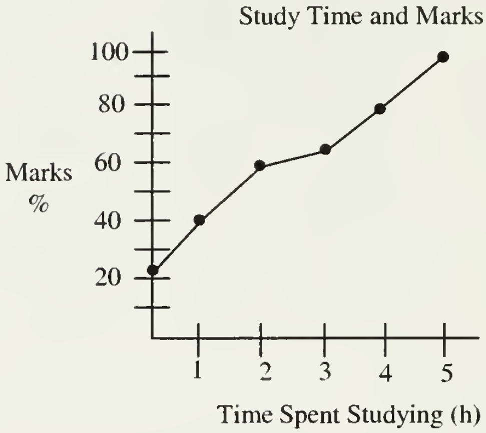
	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Recognizing and Identifying Graphs • Have students find examples of graphs in local newspapers and magazines. • Identify graphs as: – bar graphs – line graphs (broken line graphs) – circle graphs (pie graphs) – pictographs. Use of Graphs in Providing Information • What information is given? (title, labels) • How were scales, pictures, data grouping used? • Does the graph present information clearly? • Is the graph visually appealing? • What graphs best represent certain situations? – line graphs for time – bar graphs to compare quantities – circle graphs to compare parts of a whole

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	Examples of graphs.

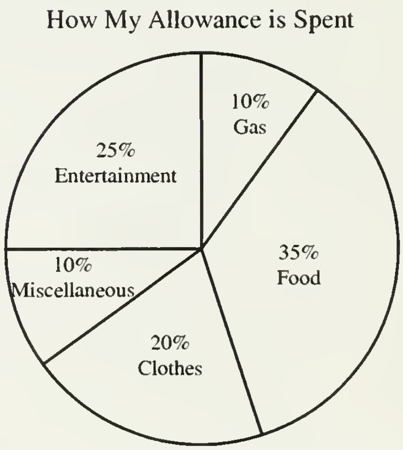
Bar graph:



Line graph:



Circle graph:



TASKS FOR INSTRUCTION AND/OR ASSESSMENT

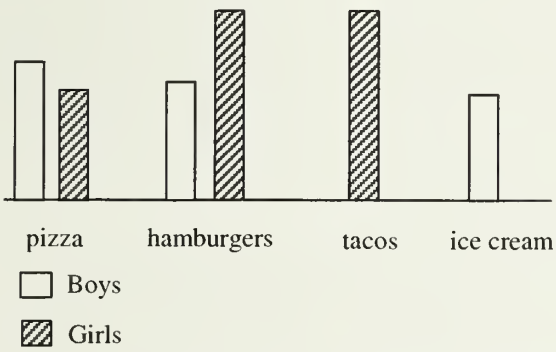
Teaching Notes

Portfolio

1. Make a scrapbook of various kinds of graphs. Collect as many as possible, and separate them into categories according to type; e.g., bar, line, circle. Discuss what conclusions can be made from each.

Paper and Pencil

1.

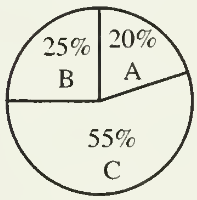


   a. What conclusions could you make from this graph?
   b. What important information is missing?
   c. Could this be true in your school?

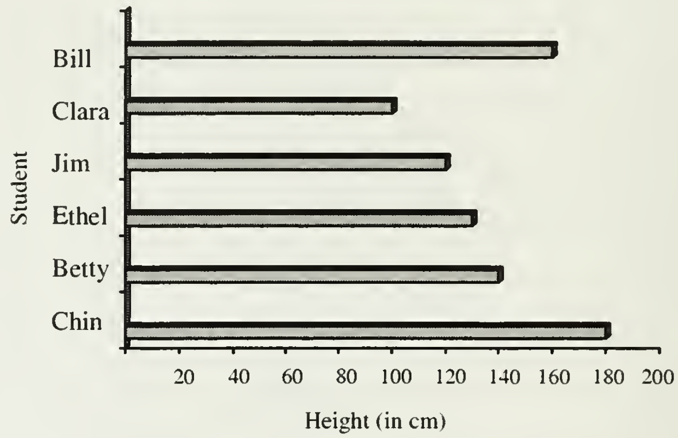
2. Reading graphs for information.

   a. What percentage of the students received a mark of A? B? C?
   b. What fraction of students received a mark of B? C?
   c. If there were 80 students, how many received a B?

Grade 10 Mathematics Marks



	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	3. Class "XY" Heights



a. How tall is Bill?
b. Who is about 130 cm tall?
c. Who might be a future basketball player?

Interview/Discussion/Journal

1. Give students three or four graphs, and ask them to select the one that best represents a given situation. Discuss why one is better than the others.

2. Given a partial graph; e.g., title, label or scale missing, discuss what the graph describes and justify your responses.

3. Have students describe a situation when it is best to use each of the following graphs:
   a. bar graph
   b. line graph
   c. circle graph
   d. pictograph.

STRAND: STATISTICS AND PROBABILITY (DATA ANALYSIS)

GENERAL OUTCOME

Analyze experimental results expressed in two variables.

SPECIFIC OUTCOMES

2. Create scatterplots for discrete and continuous variables. [C, V] (9–2)
3. Interpret a scatterplot to determine if there is an apparent relationship. [E, R] (9–3)

MANIPULATIVES

- Grid paper
- Ruler

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Addison-Wesley Mathematics 10*, pp. 448–449
- *Interactions 9*, pp. 8–10, 295
- *Mathpower 9*, pp. 330–333
- *Minds on Math 9*, pp. 66–70
- *TLE 9*, Scatter Plots, Student Refresher pp. 96–97, Teacher's Manual pp. 204–211

Previously Authorized Resources
- *Math Matters: Book 2*, pp. 267–269

TECHNOLOGY CONNECTIONS

- TI-83 graphing calculator (optional)
- Spreadsheet software

<table>
<tr><th></th><th>INSTRUCTIONAL STRATEGIES/SUGGESTIONS</th></tr>
<tr><td>Teaching Notes

Wherever possible, use data that is relevant to student interest or can be obtained by measuring.</td><td>Scatterplots are graphs used to analyze trends or relationships between bivariate data.

It is useful to present data to students in the form of a problem and then have students form a hypothesis of what they expect the relationship to be. Scatterplots can be created to test their hypothesis and to observe any trends or correlations in the data. From these observations, predictions can be made; e.g., Is there a relationship between a student's height and his/her mass? Is there a relationship between a student's month of birth and shoe size?

Collecting Data and Creating a Scatterplot

What is the relationship between the number of students and the length of time it takes all classmates to introduce themselves?

Have students gather data, using the following procedure.</td></tr>
</table>

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

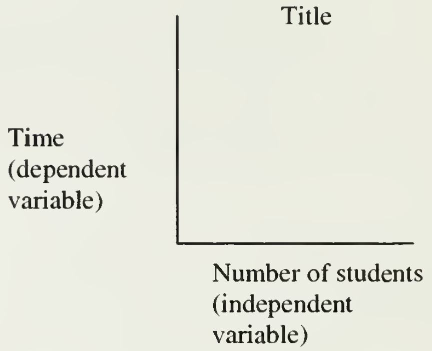
Alternative names for the independent variable include manipulated and input.

Alternative names for the dependent variable include responding and output.

1. Start the timer—the time must be continuous.
2. The first student stands and states: My name is ___________ .
   I like to ______________ .
3. The first student sits down and the next student completes part 2.
4. The continuous/cumulative time should be recorded after each student finishes.

number of students	time

5. Create a scatterplot.



Optional: Creating a scatterplot using a graphing calculator (instructions given are for a TI-83 graphing calculator)

Assume the following data set was obtained from the activity above:

Number of students	30	25	6	10	15
Time in seconds	85	75	10	20	35

Ensure that all equations are turned off:

“Y=” Delete all equations using the “DELETE” button.

OR

Toggle down to each = sign and hit “ENTER” to ensure the = sign is not highlighted.

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

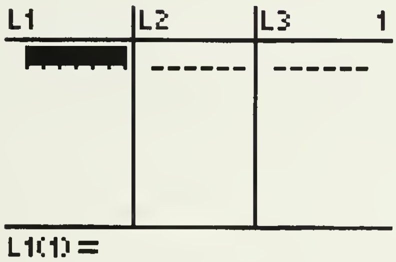
Spreadsheet software can also be used to create scatterplots and lines of best fit.

Ensure that all "stat plot" are off:
"2nd" "Y=" "4" "ENTER"

Turn only 1 stat plot on:
"2nd" "Y=" "1" "ENTER"

Ensure that all lists are free:
"2nd" "+" "4" "ENTER"

Enter data into lists:
"STAT" "1" "ENTER"



It is easier for students if they type the independent data into L1 and the dependent data into L2. This will match the calculator defaults.

In L1, type in the independent data (the number of students).
In L2, type in the dependent data (time).

Teaching Notes

Setting the minimum and maximum values manually for *x* and *y*, as well as the scale, are important skills for students to have for later mathematics courses.

INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Create the scatterplot:

"2nd" "Y=" "1"

Toggle down using the down arrow to "Type" and select scatterplot
Ensure that Xlist is L1 (independent)
Ensure that Ylist is L2 (dependent)
Select a marker using the down arrow.

"GRAPH"

To fit the data automatically: "ZOOM" "9"
To fit the data using windows: "WINDOWS" and set the windows using appropriate values.

Interpreting a Scatterplot

The interpretation of a scatterplot is important to highlight trends or relationships within the data. There are three basic trends/correlations:

1. no relationship (zero correlation)
2. a positive relationship (shows an upward trend)
3. a negative relationship (shows a downward trend).

The scatterplot from the above exercise can be analyzed for its trend. Make an overhead of the scatterplot, and place a clear plastic ruler in the direction the data points go.

Does the data represent: an upward trend?
a downward trend?
no trend at all?

How does this observed trend relate to your hypothesis?

TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

Paper and Pencil/Technology

1. Is there a relationship between the age of 14 randomly chosen students and the number of days they were absent?

Age:	23	24	23	27	26	26	26	26	27	28	26	25	37	24
Absent:	8	6	11	7	11	8	10	11	10	8	5	8	9	10

2. Is there a relationship between the mass of a car and the fuel consumption (FC), in litres per 100 km, for combined city and highway driving?

Mass (kg):	700	1000	1100	1200	1300	1500	1700	1800
FC (L/100 km):	5.0	6.5	7.5	9.0	11.0	12.0	12.0	12.0

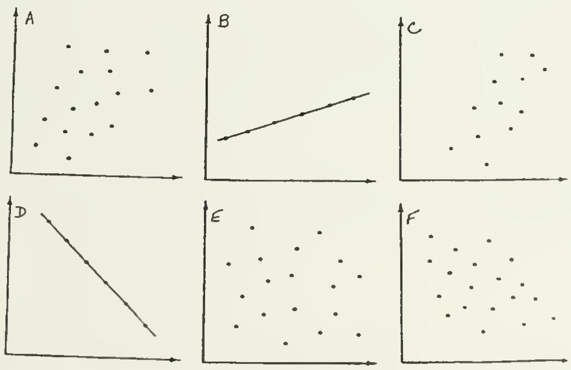
Predict the combined fuel consumption for a car with a mass of 1400 kg.

3. Does the mass of a bike affect its jumping height?

Mass (kg):	10.9	10.8	7.8	9.9	9.0	8.0	10.3	8.6	10.7
Height (cm):	24.3	24.5	26.3	24.9	25.6	26.2	24.8	26.0	24.7

John has entered a competition where his bike is expected to be able to jump at least 25 cm. To the nearest tenth of a kilogram, what is the maximum mass the bike should be?

4. There are six scatterplots shown below labelled A to F.
   a. Which of these show a positive correlation?
   b. Which of these show a negative correlation?
   c. Which of these has a correlation coefficient that is closest to zero? Describe what a correlation coefficient of zero tells us about the variables.
   d. The correlation coefficients of the scatterplots below are as follows: 1, 0.75, 0.5, 0, –0.5, –1. Identify the scatterplot associated with each of these correlation coefficients.



	TASKS FOR INSTRUCTION AND/OR ASSESSMENT
Teaching Notes	5. State whether you expect a strong positive, strong negative, weak positive, weak negative or no correlation between the variables in each case. a. waist size and the time to run a long-distance race b. IQ and shoe size c. price of an airline ticket and the distance travelled by the airplane d. hours of homework and grade point average e. number of absences and test marks f. age and test marks g. penalty minutes and number of goals scored h. ski sales and air temperature **Journal/Interview** 1. a. Describe what correlation means. b. Compare and contrast the three main types of correlation. c. How are scatterplots used to analyze data? d. Describe how predictions can be made about the data. 2. If a strong positive correlation exists between the amount that people smoke and their chance of getting cardiovascular disease (CVD), does this prove that smoking causes CVD? Discuss.

STRAND: STATISTICS AND PROBABILITY (DATA ANALYSIS)

GENERAL OUTCOME Analyze experimental results expressed in two variables.

SPECIFIC OUTCOME 4. Determine the line of best fit from a scatterplot for an apparent linear relationship by inspection. [E, PS] (9–4)

MANIPULATIVES
- Ruler
- Grid paper

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources
- *Addison-Wesley Mathematics 10*, pp. 162–163, 214, 232, 450–451
- *Interactions 9*, pp. 11–12, 153, 295
- *Mathpower 9*, pp. 334–339, 341
- *Minds on Math 9*, pp. 71–75
- *TLE 9*, Line of Best Fit, Student Refresher pp. 100–101, Teacher's Manual pp. 212–215

Previously Authorized Resources
- *Math Matters: Book 2*, pp. 270–273

TECHNOLOGY CONNECTIONS TI-83 graphing calculator (optional)

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	see below

Teaching Notes

Line of Best Fit—The line that best illustrates the relationship between the variables.

A clear or transparent ruler works best for drawing a line of best fit by inspection. By covering the majority of the points, the line of best fit will show the trend. Put the ruler along a line that shows the trend such that the following is seen:
- approximately the same number of points above or below (covered and uncovered)
- approximately equal distances from these points to the line.

Use the following data to determine the line of best fit.

a.

mass of car (kg)	700	1000	1100	1200	1300	1500	1700	1800
fuel consumption (L/100 km)	5.0	6.5	7.5	9.0	11.0	12.0	12.0	12.0

b.

mass of bike (kg)	10.9	10.8	7.8	9.9	9.0	8.0	10.3	8.6	10.7
height of jump (cm)	24.3	24.5	26.3	24.9	25.6	26.2	24.8	26.0	24.7

c. students' heights and arm spans

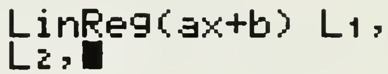
What predictions can you make based on the line of best fit?

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	

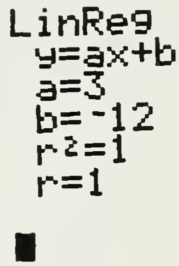
Optional: Finding a line of best fit using a graphing calculator (instructions given are for a TI-83 graphing calculator)

To select a linear regression:
"STAT" "CALC" "4"

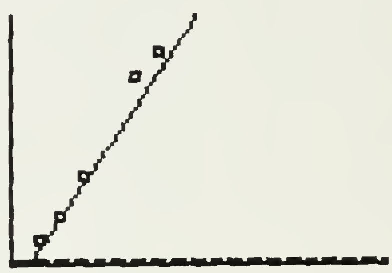
To specify which lists data is stored in:
"2nd" "1" "," "2nd" "2" "," "VARS" "Y-VARS"



To store the equation in "Y=" and to display the equation:
"," "VARS" "Y-VARS" "ENTER" "ENTER" "ENTER"



To view the scatterplot with the line of best fit:
"GRAPH"



	TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

Journal/Interview

1. Describe how you would draw a line of best fit and how to use it to make predictions.

2. How does a line of best fit relate to the trend of the data?

Portfolio

1. Find examples of data with which you would use a line of best fit; e.g., sports statistics. Plot the data and determine the line of best fit.

Paper and Pencil/Technology

1. Plot the data below and draw a line of best fit.

Mass (kg)	100 m time (s)	Mass (kg)	100 m time (s)
42	12.3	47	14.0
43	12.4	48	14.7
44	12.9	46	13.5
41	12.0	45	13.1
40	12.0	51	15.6

2. Below is an approximate comparison of the actual age of a dog and its equivalent age in "human years." Plot the data and draw a line of best fit.

Age of Dog (years)	1	2	4	6	8	10	14	18	20	21
Approximate Human Age (years)	15	24	32	40	48	56	72	90	94	101

STRAND: STATISTICS AND PROBABILITY (DATA ANALYSIS)

GENERAL OUTCOME

Analyze experimental results expressed in two variables.

SPECIFIC OUTCOME

5. Draw and justify conclusions from the line of best fit by:
   - interpolation
   - extrapolation.

   [C, R] (9–5)

MANIPULATIVES

- Ruler
- Grid paper

SUGGESTED LEARNING RESOURCES

Currently Authorized Resources

- *Addison-Wesley Mathematics 10*, pp. 162–163, 214, 232
- *Interactions 9*, pp. 11–16, 295
- *Mathpower 7*, p. 212
- *Mathpower 9*, pp. 334–339
- *Minds on Math 7*, pp. 242–243
- *Minds on Math 9*, pp. 71–75
- *TLE 9*, Drawing Conclusions, Student Refresher pp. 102–103, Teacher's Manual pp. 216–219

Previously Authorized Resources

- *Journeys in Math 9*, pp. 310–312
- *Math Matters: Book 2*, pp. 151, 152

TECHNOLOGY CONNECTIONS

TI-83 graphing calculator (optional)

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS
Teaching Notes	A line of best fit is an important tool to predict values within the data set (interpolation) and outside the data set (extrapolation).

Terminology

Interpolate—To estimate (or calculate) a value between two values that are already known.

Extrapolate—To estimate (or calculate) a value, by using the established pattern and going beyond known values.

Use the data below to create a scatterplot and line of best fit and to answer the following questions.

Number of students	30	25	6	10	15
Time to introduce in seconds	85	75	10	20	35

1. How long will it take five students to introduce themselves?

	INSTRUCTIONAL STRATEGIES/SUGGESTIONS

Teaching Notes

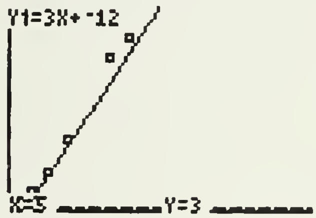
2. Did you interpolate or extrapolate to obtain your result in question 1?
3. How long will it take 60 students to introduce themselves?
4. Did you interpolate or extrapolate to obtain your result in question 3?
5. If your data set included five students, why is the time in the data set the same as or different from the predicted value obtained from using the line of best fit?
6. If it took 70 seconds for a group to introduce themselves, estimate how many people were in that group?
7. Did you interpolate or extrapolate to obtain your result in question 6?

Optional: Interpolating and extrapolating values from a scatterplot, using a graphing calculator (instructions given are for a TI-83 graphing calculator)

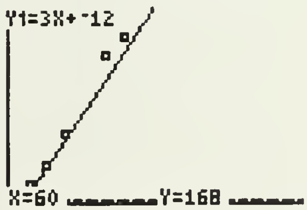
This is beyond the scope of the Mathematics Preparation 10 curriculum.

The view screen should indicate the scatterplot with the line of best fit. To predict a *y* value (dependent value), given an *x* value (independent value):

To answer question 1 above:
"2nd" "TRACE" "1" "5" "ENTER"



To answer question 3 above:
"2nd" "TRACE" "1" "60" "ENTER"



Note: The view screen may show "err:invalid".
If this occurs, the value typed in for *x* is out of the window settings. Go back to "WINDOWS" and adjust the maximum and minimum values for *x* and *y* accordingly. Then repeat the calculator sequence above to predict a time for *x* students.

	TASKS FOR INSTRUCTION AND/OR ASSESSMENT

Teaching Notes

Journal/Interview

1. What is the difference between interpolation and extrapolation?

2. In what data situations would you use interpolation? extrapolation?

Paper and Pencil/Technology

1. Some feedlot calves gain three pounds per day. A 650 pound calf is placed in a pen and weighed every 5 days. The following weights are recorded. How long will it take the calf to reach 975 pounds?

Days	Weight (pounds)
0	650
5	664
10	684
15	695
20	710

2. Pairs of American bald eagles have increased in the last forty years as indicated in the chart below. Graph the data and find a line of best fit. From the line of best fit, predict:
   a. the number of bald eagle pairs in 1982 and 1994
   b. the number of bald eagle pairs in 2010.

   Would it be better to use a curve of best fit for this data? Explain your reasoning.

Year	Estimated Number of Bald Eagle Pairs
1963	400
1974	800
1981	1200
1984	1800
1988	2500
1991	3400
1993	4000
1996	5000
1999	5800

REFERENCES

General References

Manitoba Education and Training. *Grades 5 to 8 Mathematics: A Foundation for Implementation.* Winnipeg, MB: Manitoba Education and Training, 1997.

Manitoba Education and Training. *Senior 1 Mathematics: A Foundation for Implementation.* Winnipeg, MB: Manitoba Education and Training, 1997.

Nova Scotia Department of Education and Culture. *Atlantic Canada Mathematics Curriculum: Grade 7.* Halifax, NS: Nova Scotia Department of Education and Culture, 1999.

Nova Scotia Department of Education. *Atlantic Canada Mathematics Curriculum: Grade 8.* Halifax, NS: Nova Scotia Department of Education, 1999.

Nova Scotia Department of Education. *Atlantic Canada Mathematics Curriculum: Grade 9 Implementation Draft.* Halifax, NS: Nova Scotia Department of Education, 1999.

Province of British Columbia Ministry of Education. *Mathematics 8 to 10: Integrated Resource Package.* Victoria, BC: Province of British Columbia Ministry of Education, 1996.

Currently Authorized Resources

Adomeit, Joanne et al. *The Learning Equation: Mathematics 7. Student Refresher.* Toronto, ON: ITP Nelson, 1998.

Adomeit, Joanne et al. *The Learning Equation: Mathematics 7. Teacher's Manual.* Toronto, ON: ITP Nelson, 1999.

Adomeit, Joanne et al. *The Learning Equation: Mathematics 8. Student Refresher.* Toronto, ON: ITP Nelson, 1999.

Adomeit, Joanne et al. *The Learning Equation: Mathematics 8. Teacher's Manual.* Toronto, ON: ITP Nelson, 1999.

Alexander, Robert and Brendan Kelly. *Addison-Wesley Mathematics 10.* Don Mills, ON: Addison-Wesley, 1998.

Alexander, Robert et al. *Minds on Math 7.* Don Mills, ON: Addison-Wesley Publishers Limited, 1996.

Alexander, Robert et al. *Minds on Math 8.* Don Mills, ON: Addison-Wesley Publishers Limited, 1996.

Alexander, Robert et al. *Minds on Math 9.* Don Mills, ON: Addison-Wesley Publishers Limited, 1996.

Allison, Paula et al. *The Learning Equation Mathematics 9. Student Refresher.* Toronto, ON: ITP Nelson, 1998.

Allison, Paula et al. *The Learning Equation Mathematics 9. Teacher's Manual.* Toronto, ON: ITP Nelson, 1998.

Boucher, Randy et al. *The Learning Equation: Mathematics 10. Student Refresher.* Toronto, ON: ITP Nelson, 1998.

Boucher, Randy et al. *The Learning Equation: Mathematics 10. Teacher's Manual.* Toronto, ON: ITP Nelson, 1998.

Elchuck, Larry et al. *Interactions 7.* Scarborough, ON: Prentice Hall Ginn Canada, 1996.

Elchuck, Larry et al. *Interactions 8.* Scarborough, ON: Prentice Hall Ginn Canada, 1996.

The Geometer's Sketchpad (Macintosh Version 3.0). Berkeley, CA: Key Curriculum Press, 1995.

The Geometer's Sketchpad (Windows Version 3.03). Berkeley, CA: Key Curriculum Press, 1995.

Hope, Jack et al. *Interactions 9.* Scarborough, ON: Prentice Hall Ginn Canada, 1996.

Knill, George et al. *Mathpower 7* (Western Edition). Toronto, ON: McGraw-Hill Ryerson Limited, 1996.

Knill, George et al. *Mathpower 8* (Western Edition). Toronto, ON: McGraw-Hill Ryerson Limited, 1996.

Knill, George et al. *Mathpower 9* (Western Edition). Toronto, ON: McGraw-Hill Ryerson Limited, 1996.

Knill, George et al. *Mathpower 10* (Western Edition). Toronto, ON: McGraw-Hill Ryerson Limited, 1998.

Previously Authorized Resources

Bober, William C. et al. *Journeys in Math 9.* Ginn Publishing Canada Inc., 1988.

Connelly, Ralph D. et al. *Journeys in Math 8.* Ginn Publishing Canada Inc., 1987.

Ebos, Frank et al. *Math Matters: Book 2.* Scarborough, ON: Nelson Canada, 1989.

Kelly, Brendan et al. *Mathematics 9.* Don Mills, ON: Addison-Wesley, 1981.

Additional Resources

Alberta Education. *Kindergarten to Grade 9 Mathematics Resources: Annotated Bibliography.* Edmonton, AB: Alberta Education, 1997.

Burns, Marilyn. *Spaghetti and Meatballs for All!: A Mathematical Story.* New York, NY: Scholastic, 1997.